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ཚལ་ལྷན་ཁག། རགས་ཚལ་དང་སྤོང་ཀ་འབྲས་
རྟོག་ལས་ཁུངས།



Royal Government of Bhutan
Ministry of Agriculture and Forests
Department of Forests and Park Services

SPECIES SPECIFIC VOLUME EQUATION TO ESTIMATE MERCHANTABLE VOLUME

Tsuga dumosa

Forest Resources Management Division
Department of Forest and Park Services
Ministry of Agriculture and Forests

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Species specific volume equation to
estimate merchantable volume

Tsuga dumosa

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Table of Contents

1. Summary.....	1
2. Introduction.....	2
3. Volume Calculation.....	3
4. The Dataset used for modeling volume of <i>Tsuga dumosa</i>	4
4.1 Summary descriptive statistics of <i>Tsuga dumosa</i> dataset	4
5. Fitting the models.....	5
6. Summary Plots.....	6
7. Models and results.....	7
7.2 Model 1 - Volume with diameter at breast height (DBH) as predictor.....	7
7.2 Model 2 - Volume with diameter at breast height (DBH) as predictor, with varFixed	8
7.3 Model 3- Volume with diameter at breast height (DBH) as predictor, with varPower.....	9
7.4 Model 4 - Volume with diameter at breast height (DBH) as predictor, with varConstPower ...	10
7.5 Model 5 - Volume with basal area (BA) as predictor.....	11
7.6 Model 6 - Volume with basal area (BA) as predictor, with varFixed.....	12
7.7 Model 7 Volume with basal area (BA) as predictor, with varPower.....	13
7.8 Model 8 – Volume with basal area (BA) as predictor, with varConstPower.....	14
7.9 Model 9 – Volume with square of diameter at breast height * height (DBH2H) as predictor..	15
7.10 Model 10 – Volume with square of diameter at breast height * height (DBH2H) as predictor, with varFixed.....	16
7.11 Model 11– Volume with square of diameter at breast height * height (DBH2H) as predictor, with varPower.....	17
7.12 Model 12 –Volume with square of diameter at breast height * height (DBH2H) as predictor, with varConstPower	18
7.13 Model 13 – Volume with basal area * height (BAH) as predictor.....	19
7.14 Model 14 – Volume with basal area * height (BAH) as predictor, with varFixed	20
7.15 Model 15– Volume with basal area * height (BAH) as predictor, with varPower.....	21
7.16 Model 16 – Volume with basal area * height (BAH) as predictor, with varConstPower	22
1. Model evaluation using AIC and BIC values.....	23
8. Selected Models	25
9. Demonstration of use of the selected best fit model.....	25
10. Model Performance.....	28
11. Limitations of the model.....	30
12. Conclusion.....	31
13. Acknowledgement.....	32
14. References.....	33
15. Annexure – Dataset for <i>Tsuga dumosa</i>	35

1. Summary

The volume equation developed in this study will predict the merchantable volume of *Tsuga dumosa*. The standards of merchantability adopted for this study to develop merchantable log volume equation are trees of 10 cm and above diameter at breast height (dbh) and the sections up to 10 cm top diameter over the bark.

A total of 16 models were fitted. First 4 models were fitted with volume as a function of diameter at breast height (DBH), while models 5 – 8 were fitted with basal area (BA) as the predictor variable. With product of squared diameter at breast height and height (DBH²H) as predictor variable, 4 models, namely the models 9 – 12 were fitted. The last four models, 13 -16 were fitted with product of basal area and height (BAH) as the predictor.

The initial plots of response (volume) variables and predictor (DBH, BA, DBH²H and BAH) variables clearly indicated presence of heteroscedasticity, which has been modeled using variance functions (varFixed, varPower and varConstPower) in gls () function of nlme package.

Of the sixteen, two models viz model 7 for those models which were fitted without height and model 15 for those models which were fitted with height as predictors, have been selected as the best fit models. The model 7 had AIC and BIC values of 116 and 126 respectively, whereas the model 15 had AIC and BIC values of 26 and 37 respectively. Lower the AIC and BIC values, better the fit of the model.

The performance of the selected models was assessed by comparing the actual volume with the volumes predicted by two selected models for each tree. From the assessment, both models were observed to be performing equally well. However, by virtue of having the lowest AIC and BIC values and also owing its better prediction of the volume, the model 15 is considered to be the best fit model amongst 16 models fitted in this study.

2. Introduction

The volume equations, developed during pre-investment survey (PIS) carried out between 1974-81 predict total tree volume, and not the merchantable volume of trees. The recent change of policy of the Department of Forests and Park Services to allot timber for rural house construction in the form of log volume instead of allotting by number of trees as was once practiced, has necessitated development of merchantable log volume equation.

Therefore, standards of merchantability adopted for this study to develop merchantable log volume equation are trees of 10 cm and above diameter at breast height (dbh) and the sections up to 10 cm top diameter over the bark.

As was done for PIS exercise to develop volume equation, this study ignores/does not consider the volume of foliage and branches for the purpose of calculating the merchantable volume. This decision stems from the objective, which is to estimate merchantable log volume. Moreover, branches are rarely used as timber (at least in Bhutan) and are mostly used for firewood.

The sample trees for this study have been felled as part of biomass equation development field work and also later from a separate field work to collect volume data. The data protocol for biomass equation development required collecting a minimum of 8 trees each from four regions of Bhutan namely, eastern, eastern central, western and western central. While, through biomass exercise data for 49 trees were collected from four regions, additional data for 18 trees have collected from western region through field work conducted between November 2018- January, 2019. Thus, total sample trees for *Tsuga dumosa* used in this study was 67.

The trees were felled at 0.3 m height from the ground at which the diameter was measured and recorded.

Then diameter at zero height (ground level) were also measured and recorded. After felling, the diameter was measured at 0.7 m from 0.3 m height (essentially making 1 m height, i.e 0.3 m + 0.7 m =1 m). Thereafter, at every meter length, the diameter was measured and recorded, thus making many 1 m length sections of log. As mentioned above the smallest top diameter considered for merchantable log volume calculation was up to 10 cm diameter over bark. Top sections below 10 cm diameter have been discarded.

3. Volume Calculation

Trees after felling are converted into different sizes of sections depending on the requirement and demand. Sections with length of 8 or more feet long are called logs and shorter ones are called sticks or bolts (Avery and Burkhart, 1994). The scaling or measuring the volume of the section is done by multiplying the length with the cross-sectional area of the section. Although they rarely form true circles, they are assumed so for the purpose of calculating cross sectional area in meter square, which is

$$\text{Cross sectional area (A)} = A = \pi r^2 = \frac{\pi D^2}{4 \times 10000} \quad (1)$$

Where **r** is radius in meters and **D** is diameter at breast height in centimeters.

From the ground level to 0.3 m height (height at which sample tree has been cut) is section I, while 0.3 m to 0.7 m is section II. The subsequent sections of 1 m length each are numbered III, IV and so on. The last section is the terminal section, whose length is equal to or less than 1 m. In this study the branch and foliage volumes have been ignored for the purpose of calculating merchantable volume.

The most commonly used formulae for calculating volume are the Huber, Newton and Smalian's formulae (Sadiq, 2006, and Goulding, 1979). Of the three commonly used volume calculation approaches or formulae, the Smalian's formula has been used to calculate volume (in m³) for this study, which is;

$$\text{Section volume (V}_s\text{)} = \frac{A+a}{2} * L \quad (2)$$

Where A = Cross sectional area in m² at large end of the section
 a = Cross sectional area in m² at small end of the section
 L = Length of the section in meter

Smalian's formula is the easiest and least expensive to apply and therefore applied to get volume for each section of the sample trees. However, for the terminal section, the following formula was used to calculate the volume;

$$\text{Terminal section volume (V}_t\text{)} = \frac{A}{3} * L \quad (3)$$

The volume for sections and terminal section for individual trees were then summed to obtain the total volume for each individual sample tree, which is;

$$\text{Volume of tree (V)} = \sum_{s=1}^n V_s + V_t \quad (4)$$

After obtaining individual tree volume (Volume.m3), it was then tabulated against the variables - height in meter (Height.m) and the diameter at breast height in centimeter (DBH.cm).

4. The Dataset used for modeling volume of *Tsuga dumosa*

A total of 67 trees have been felled and collected data for modeling merchantable volume of *Tsuga dumosa*. While, data for 49 trees were collected from four regions by Biomass equation development, the additional data for 18 trees have collected from western region through field work conducted between November 2018 - January, 2019. The summary is presented below, while the detailed dataset is presented as an annexure.

4.1 Summary descriptive statistics of *Tsuga dumosa* dataset

```
> summary(td)
```

Tree_ID	Height.m	DBH.cm	Volume.m3
tde01 : 1	Min. : 7.50	Min. : 12.70	Min. : 0.04933
tde02 : 1	1st Qu.:20.85	1st Qu.: 31.50	1st Qu.: 1.02011
tde03 : 1	Median :27.30	Median : 51.50	Median : 2.55492
tde04 : 1	Mean :26.63	Mean : 53.32	Mean : 3.65648
tde05 : 1	3rd Qu.:33.20	3rd Qu.: 69.25	3rd Qu.: 5.24001
tde06 : 1	Max. :46.40	Max. : 136.00	Max. : 15.18179

BA.m2	BAH.m3	DBH2H.m3
Min. :0.01267	Min. : 0.1013	Min. : 0.129
1st Qu.:0.07825	1st Qu.: 2.0408	1st Qu.: 2.598
Median :0.20831	Median : 5.4824	Median : 6.980
Mean :0.28158	Mean : 8.3795	Mean : 10.669
3rd Qu.:0.37665	3rd Qu.: 11.9133	3rd Qu.: 15.168
Max. :1.45267	Max. : 37.0629	Max. : 47.190

5. Fitting the models

The models have been fitted in R, which is a robust statistical computing environment. It is a powerful tool which provides wide range of statistical and graphical options to explore, calculate and manage data besides modelling. It is very powerful and widely used statistical tool which is free and allows user to customize the scripts depending on desired output, which is not possible in many of the statistical softwares.

After reading in the excel files into R, we created other variables namely; basal area in square meter (BA.m2), basal area in meter times height in meter (BAH.m3) and square of the diameter in meter times height in meter (DBH2H.m3). The height in meter (Height.m) and diameter in centimeter (DBH.cm) were measured and recorded in the field.

Prior to fitting models, we explored and examined each set of data by preparing descriptive summaries that provided mean, median and range of dependent/response and independent variables. Then we plotted scatter graphs which provided sense of relationship between the dependent/response (volume) and independent/predictor variables (namely DBH.cm, BA.m2, DBH2H.m3 and BAH.m3). These graphs showed curvilinear relationship between response and predictor variables. The scatter plots also clearly revealed the presence of phenomenon, referred in statistical parlance, as heteroscedasticity, which is the increase in variation in response (volume) variable with increase in value of the predictor variables.

Therefore, we fitted the models using the `gls ()` function of the `nlme` package of R, because the `gls ()` function has the capability to model heteroscedasticity. We didn't transform the variables, mainly response variable, because transformation makes it difficult to directly interpret the relationship between response and predictor variables; and secondly to compare the AIC and BIC values among the different models, the response variables need to be identical.

The models were fitted with volume as a function of four variables;

- 1) DBH.cm,
- 2) BA.m2,
- 3) DBH2H.m3 and
- 4) BAH.m3.

For each of the variable, we fitted one simple `gls ()` function, which can be written in the following form;

$$Y = \beta_0 + \beta_1 X + \epsilon, \quad (5)$$

Where Y = Volume (V) and X = predictor variable

And then fitted 3 models with restricted natural cubic spline functions. The restricted natural cubic spline function enables better tracking of curvilinear relationship between response and predictor variables. These models introduce an additional predictor variable as part of a 3 knot-cubic spline. They take the following forms;

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon, \quad (6)$$

Where Y = Response variable, volume (V)

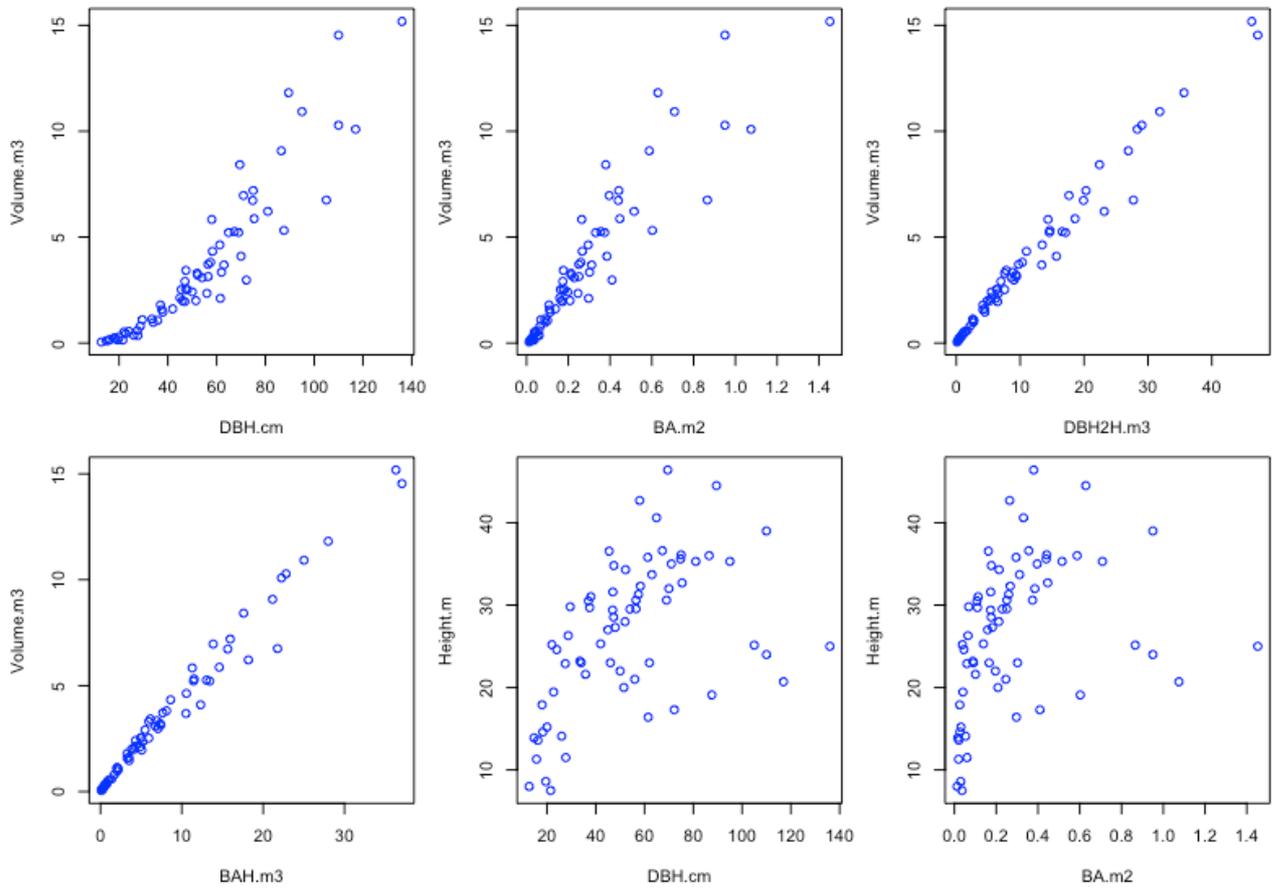
X_1 = Predictor variable

$X_2 = g(X_1)$

And $g(X_1)$ is the spline transformation of X_1 predictor variable

6. Summary Plots

Tsuga dumosa (N = 67)



7. Models and results

7.2 Model 1 - Volume with diameter at breast height (DBH) as predictor

```
> td.m1 <- gls(Volume.m3 ~ DBH.cm)
> summary(td.m1)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ DBH.cm

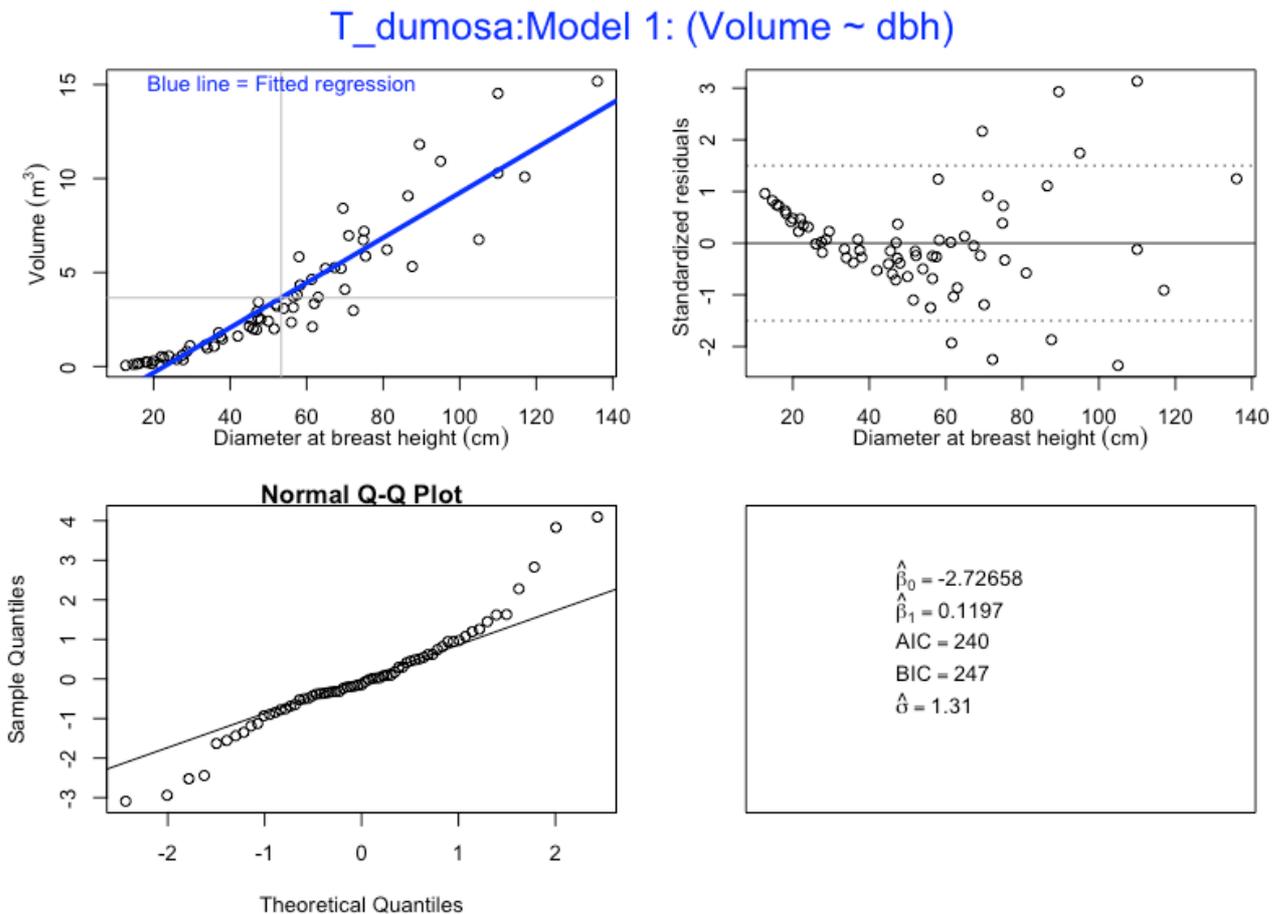
Data: NULL

	AIC	BIC	logLik
	240.2048	246.7279	-117.1024

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-2.7265775	0.3508372	-7.771633	0
DBH.cm	0.1197036	0.0058594	20.429361	0

Plot of model 1



7.2 Model 2 - Volume with diameter at breast height (DBH) as predictor, with varFixed

```
> td.m2 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints,
  na.action=na.omit, weights = varFixed(~DBH.cm))
> summary(td.m2)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ DBH.cm + DBH.cm.splinepoints

Data: NULL

	AIC	BIC	logLik
	221.3039	229.9395	-106.652

Variance function:

Structure: fixed weights

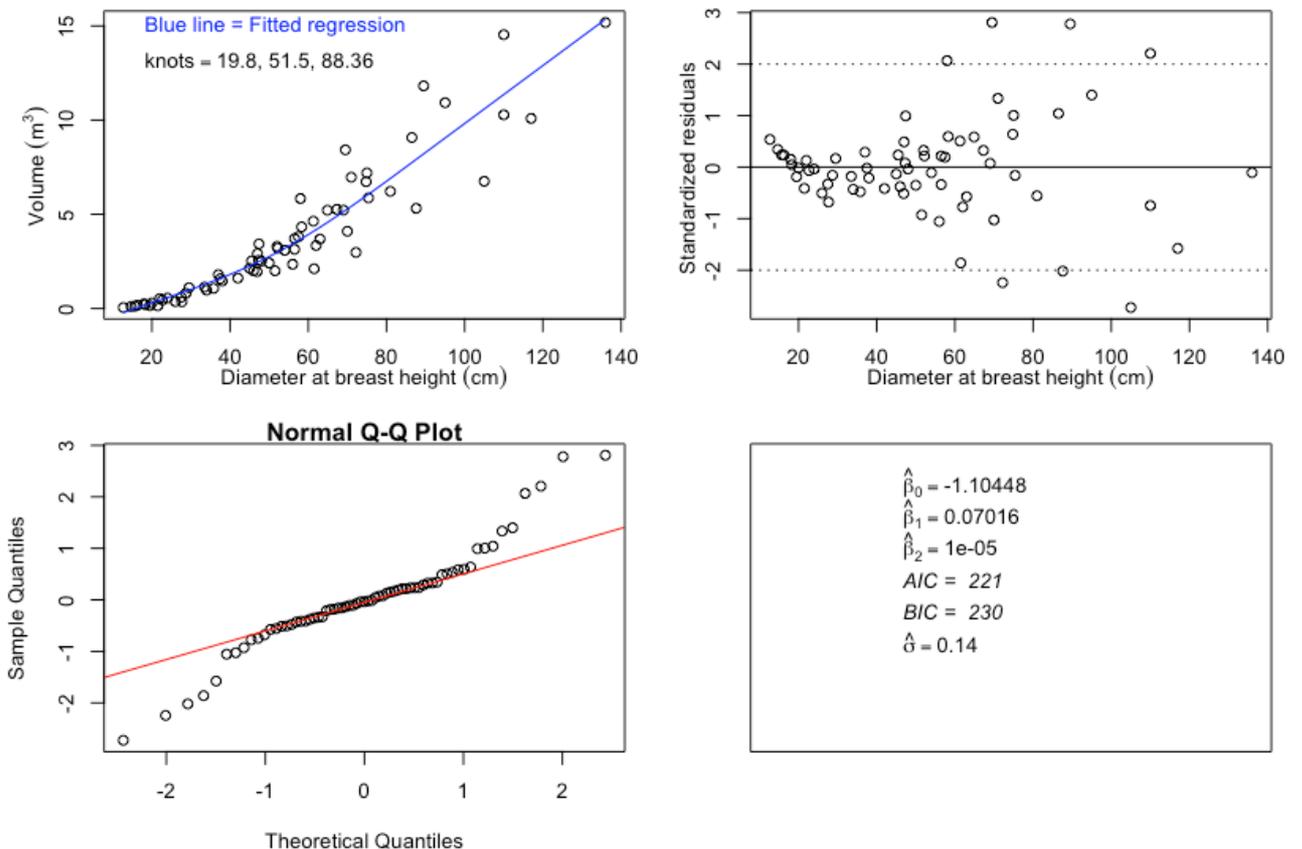
Formula: ~DBH.cm

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-1.1044763	0.29942281	-3.688685	5e-04
DBH.cm	0.0701596	0.00947484	7.404825	0e+00
DBH.cm.splinepoints	0.0000128	0.00000284	4.505198	0e+00

Plot of Model 2

T_dumosa:Model 2 : (Volume ~ dbh), Cubic spline with varFixed



7.3 Model 3- Volume with diameter at breast height (DBH) as predictor, with varPower

```
> td.m3 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints,
  na.action=na.omit, weights = varPower(form =
  ~DBH.cm))
> summary(td.m3)
```

Generalized least squares fit by REML
 Model: Volume.m3 ~ DBH.cm + DBH.cm.splinepoints
 Data: NULL
 AIC BIC logLik
 167.0114 177.8058 -78.50571

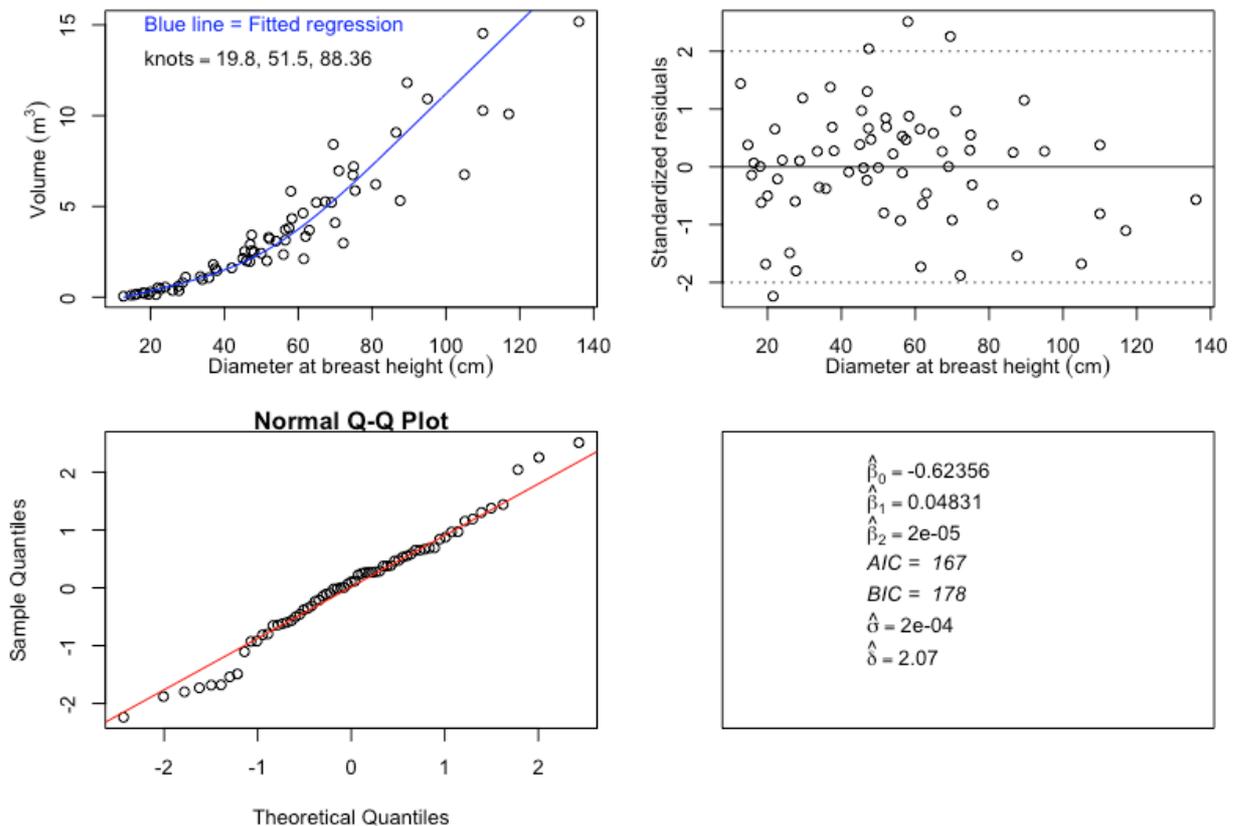
Variance function:
 Structure: Power of variance covariate
 Formula: ~DBH.cm
 Parameter estimates:
 power
 2.067238

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.6235594	0.07032189	-8.867216	0
DBH.cm	0.0483134	0.00384167	12.576144	0
DBH.cm.splinepoints	0.0000230	0.00000284	8.084054	0

Plot of Model 3

T_dumosa:Model 3: (Volume ~ dbh), Cubic spline with varPower



7.4 Model 4- Volume with diameter at breast height (DBH) as predictor, with varConstPower

```
> td.m4 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints,
  na.action=na.omit, weights = varConstPower(form =
  ~DBH.cm))
> summary(td.m4)
```

Generalized least squares fit by REML

```
Model: Volume.m3 ~ DBH.cm + DBH.cm.splinepoints
Data: NULL
      AIC      BIC    logLik
168.6158 181.5691 -78.3079
```

Variance function:

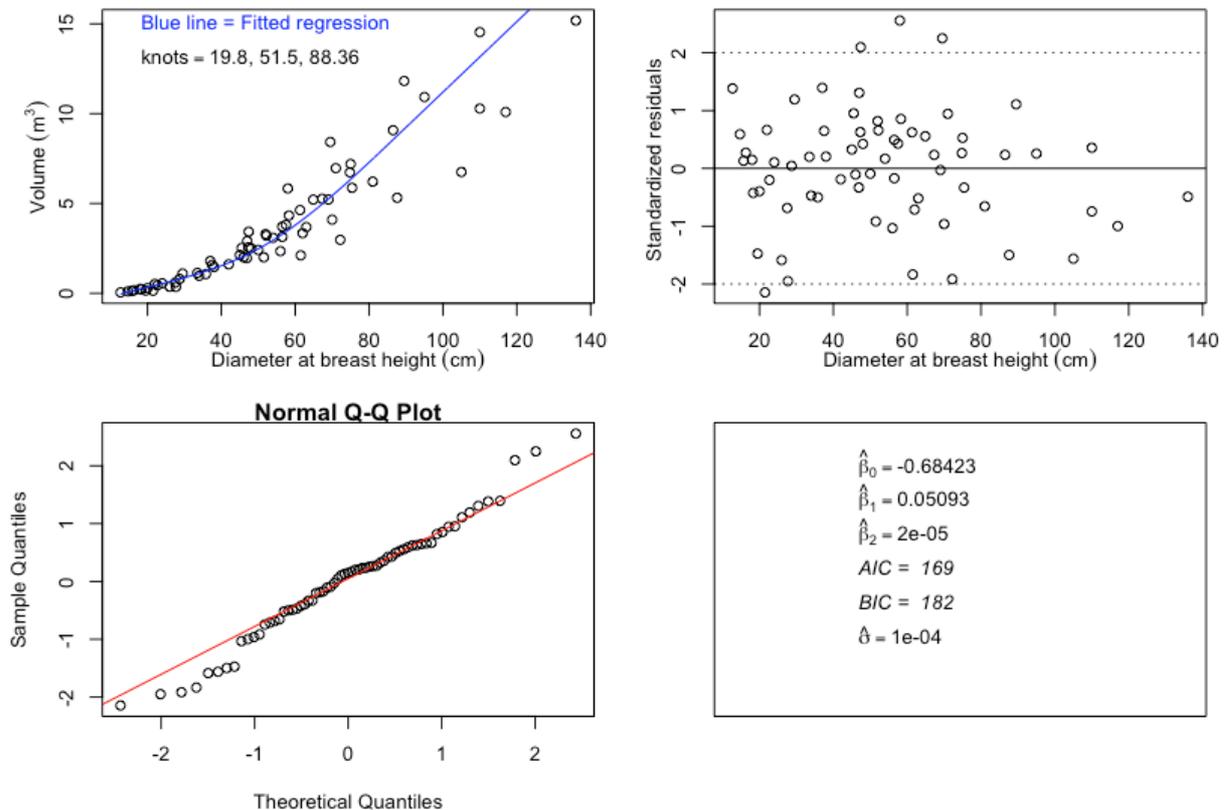
```
Structure: Constant plus power of variance covariate
Formula: ~DBH.cm
Parameter estimates:
      const      power
469.217251  2.300756
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.6842293	0.08201569	-8.342664	0
DBH.cm	0.0509251	0.00410525	12.404873	0
DBH.cm.splinepoints	0.0000223	0.00000297	7.486743	0

Plot of Model 4

T_dumosa:Model 4: (Volume ~ dbh), Cubic spline with varConstPower



7.5 Model 5 - Volume with basal area (BA) as predictor

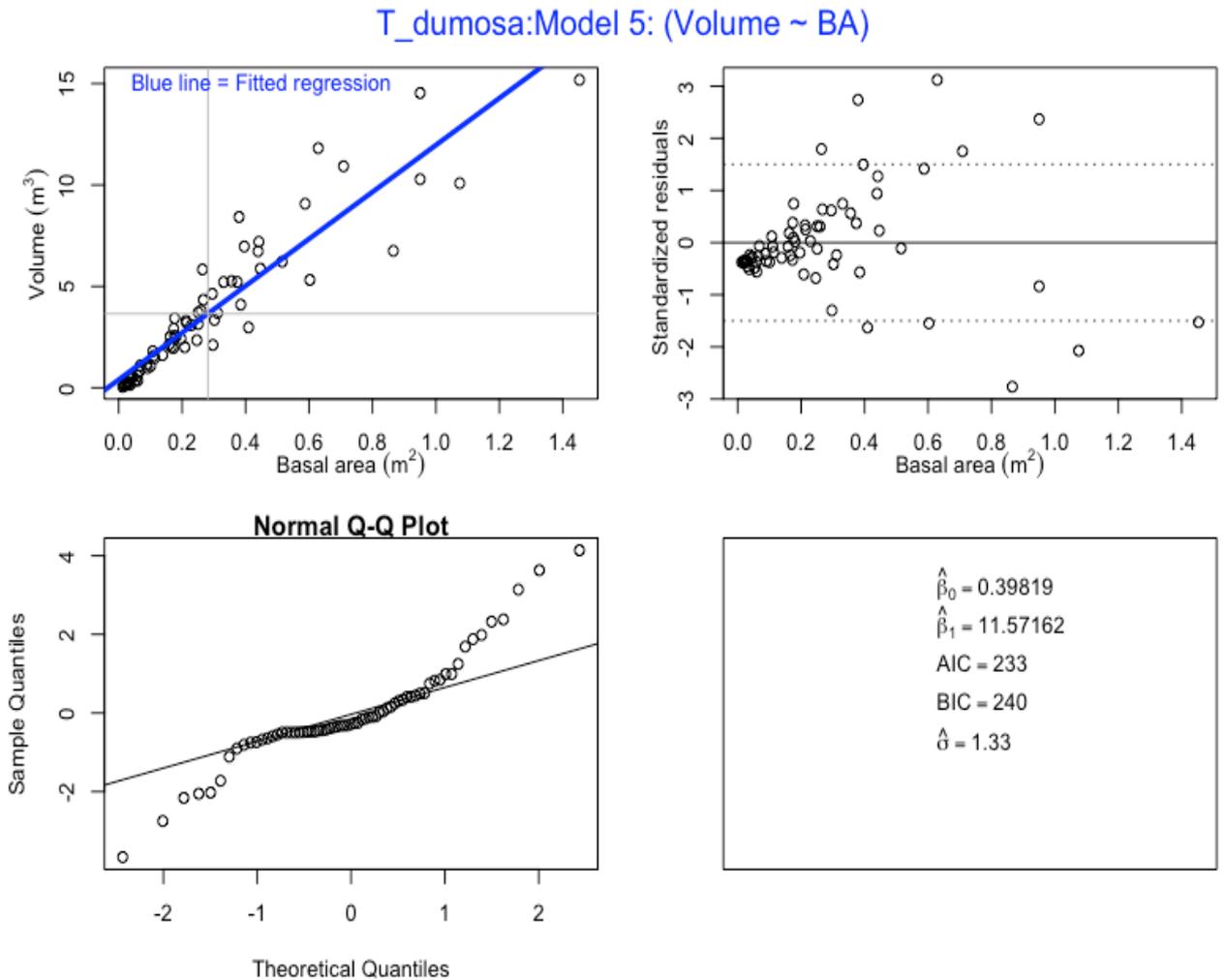
```
> td.m5 <- gls(Volume.m3 ~ BA.m2)
> summary(td.m5)
```

Generalized least squares fit by REML

```
Model: Volume.m3 ~ BA.m2
Data: NULL
      AIC      BIC    logLik
233.0569 239.58 -113.5284
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.398194	0.2293909	1.735873	0.0873
BA.m2	11.571619	0.5766051	20.068535	0.0000

Plot of Model 5

7.6 Model 6 - Volume with basal area (BA) as predictor, with varFixed

```
> td.m6<- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
              na.action=na.omit, weights = varFixed(~BA.m2))
> summary(td.m6)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ BA.m2 + BA.m2.splinepoints

Data: NULL

	AIC	BIC	logLik
	152.5056	161.1412	-72.25282

Variance function:

Structure: fixed weights

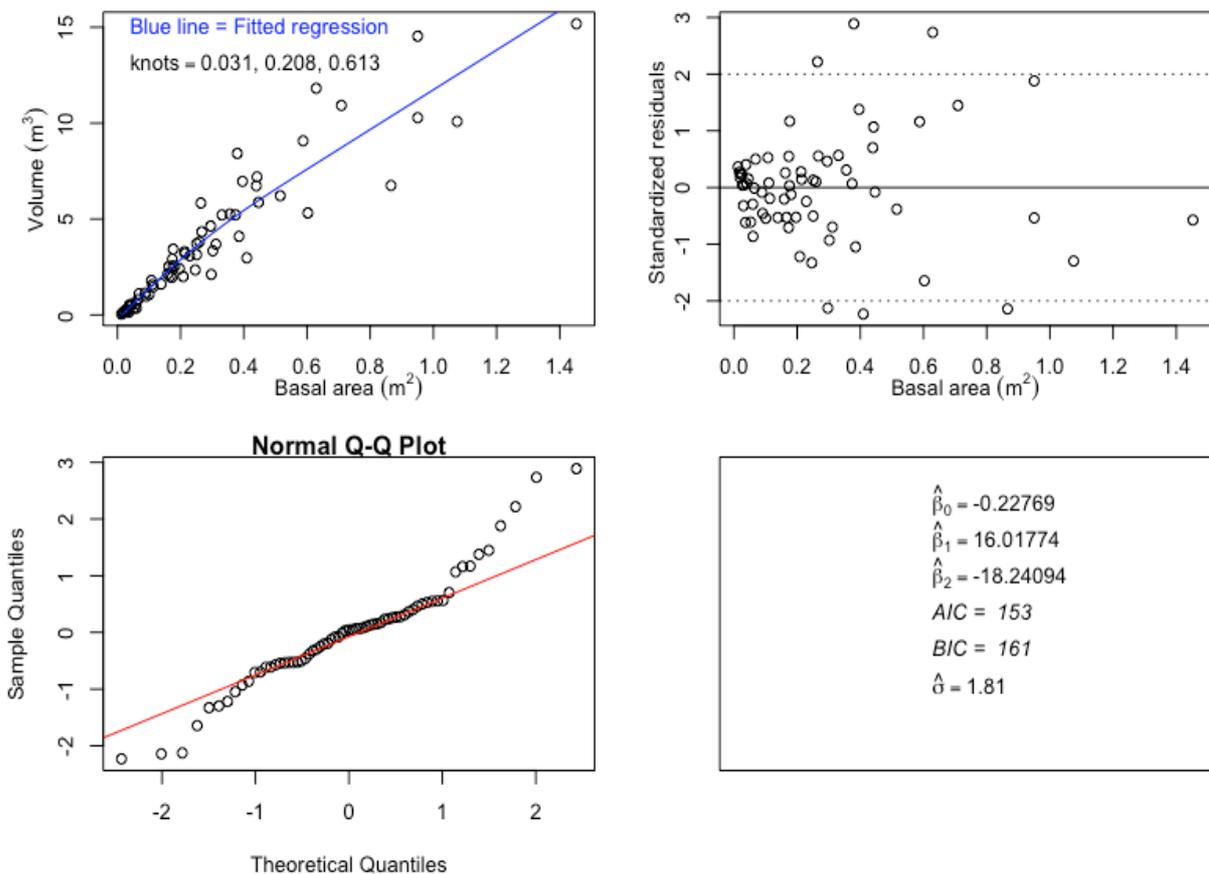
Formula: ~BA.m2

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.227692	0.092234	-2.468624	0.0162
BA.m2	16.017740	1.035761	15.464711	0.0000
BA.m2.splinepoints	-18.240943	6.005190	-3.037530	0.0034

Plot of Model 6

T_dumosa:Model 6: (Volume ~ BA), Cubic spline with varFixed



7.7 Model 7 Volume with basal area (BA) as predictor, with varPower

```
> td.m7 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
               na.action=na.omit, weights = varPower(form = ~BA.m2))
> summary(td.m7)
```

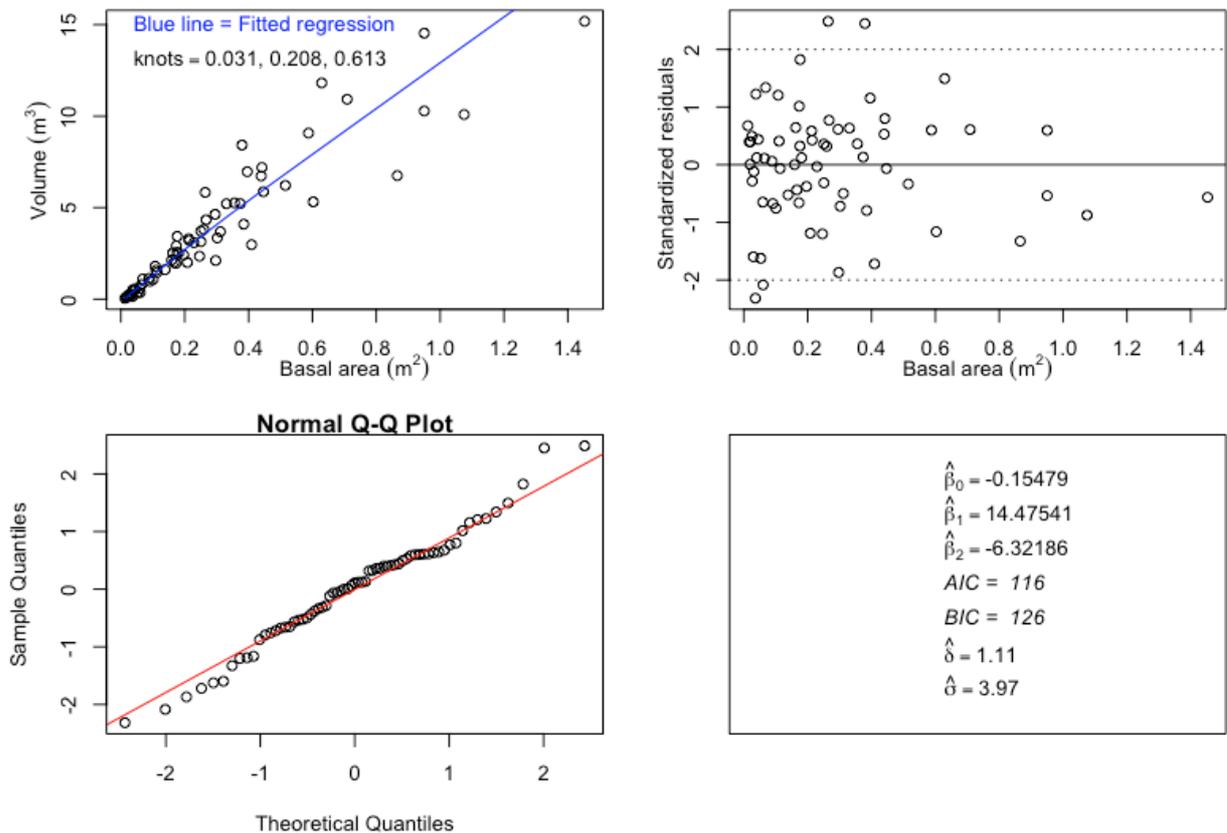
```
Generalized least squares fit by REML
Model: Volume.m3 ~ BA.m2 + BA.m2.splinepoints
Data: NULL
      AIC      BIC    logLik
115.6029 126.3973 -52.80146
```

```
Variance function:
Structure: Power of variance covariate
Formula: ~BA.m2
Parameter estimates:
  power
1.113442
```

```
Coefficients:
                Value Std.Error  t-value p-value
(Intercept)    -0.154786  0.024629  -6.284686  0.0000
BA.m2           14.475407  0.713132  20.298348  0.0000
BA.m2.splinepoints -6.321858  7.376063  -0.857078  0.3946
```

Plot of Model 7

T_dumosa:Model 7: (Volume ~ BA), Cubic spline with varPower



7.8 Model 8 – Volume with basal area (BA) as predictor, with varConstPower

```
> td.m8 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
  na.action=na.omit, weights = varConstPower(form =
  ~BA.m2))
> summary(td.m8)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ BA.m2 + BA.m2.splinepoints

Data: NULL

AIC	BIC	logLik
117.6029	130.5562	-52.80146

Variance function:

Structure: Constant plus power of variance covariate

Formula: ~BA.m2

Parameter estimates:

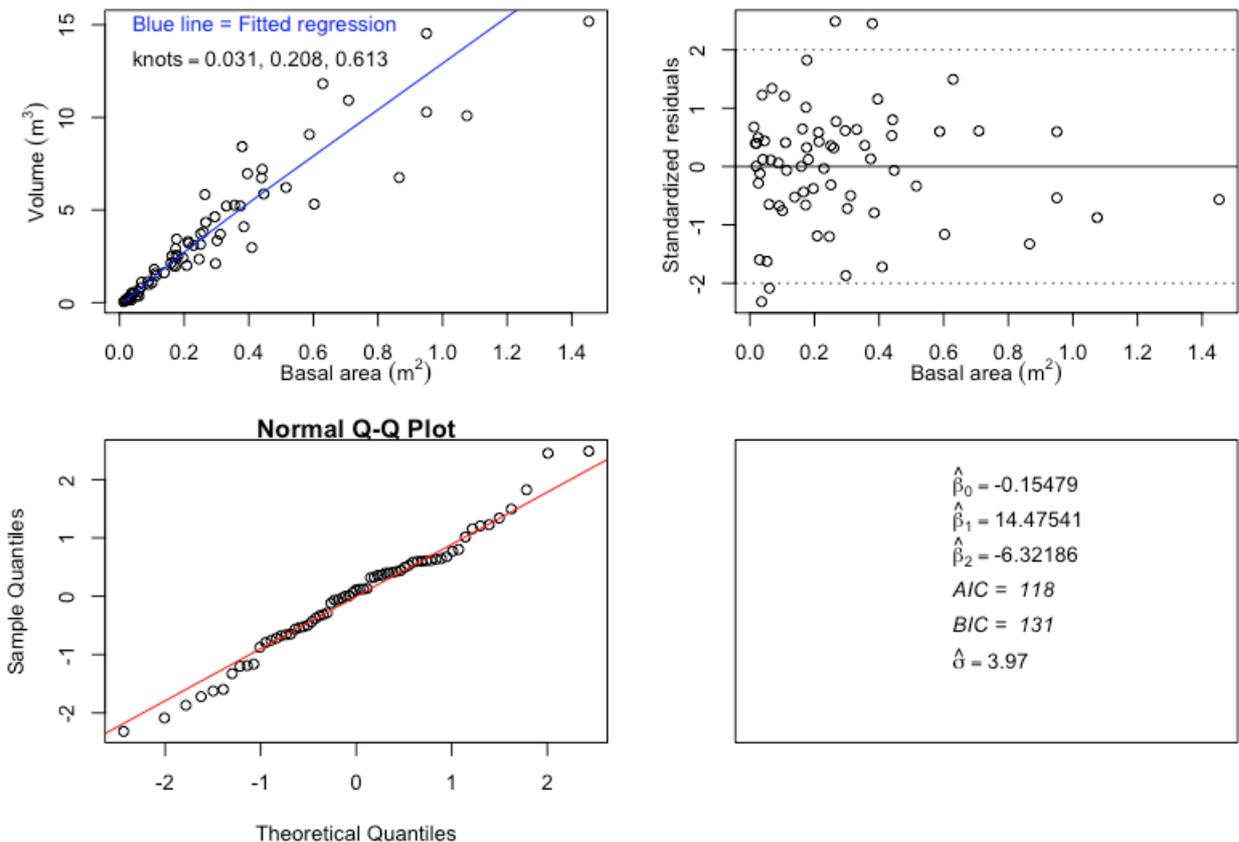
const	power
1.129914e-10	1.113442e+00

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.154786	0.024629	-6.284686	0.0000
BA.m2	14.475407	0.713132	20.298348	0.0000
BA.m2.splinepoints	-6.321858	7.376063	-0.857078	0.3946

Plot of Model 8

T_dumosa:Model 8: (Volume ~ BA), Cubic spline with varConstPower



7.9 Model 9 – Volume with square of diameter at breast height * height (DBH2H) as predictor

```
> td.m9 <- gls(Volume.m3 ~ DBH2H.m3)
> summary(td.m9)
```

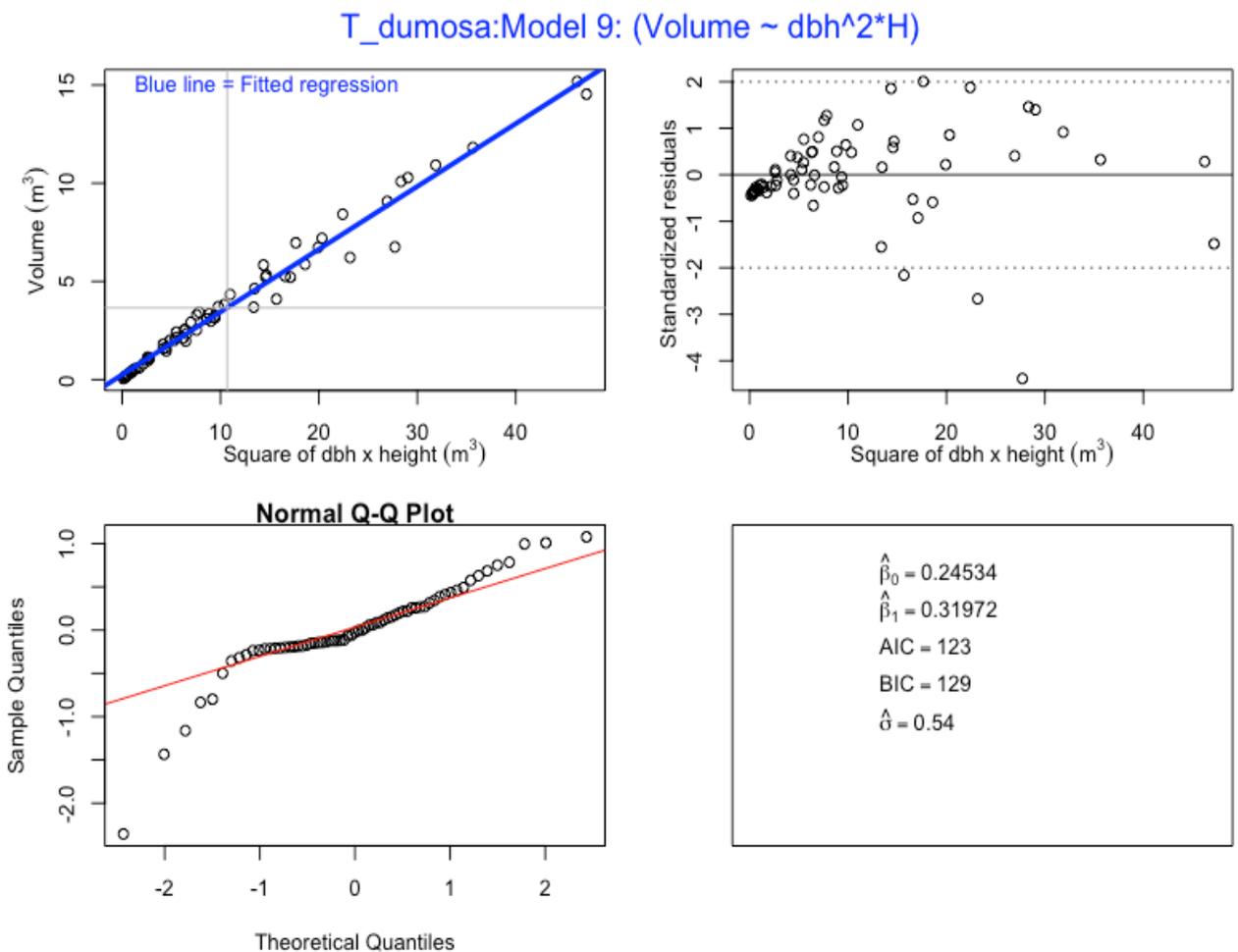
Generalized least squares fit by REML

```
Model: Volume.m3 ~ DBH2H.m3
Data: NULL
      AIC      BIC    logLik
122.9706 129.4937 -58.48529
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.2453401	0.09218293	2.66145	0.0098
DBH2H.m3	0.3197211	0.00606195	52.74232	0.0000

Plot of Model 9



7.10 Model 10 – Volume with square of diameter at breast height * height (DBH2H) as predictor, with varFixed

```
> td.m10 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,
                 na.action=na.omit, weights = varFixed(~DBH2H.m3))
> summary(td.m10)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints

Data: NULL

	AIC	BIC	logLik
	63.72767	72.3632	-27.86383

Variance function:

Structure: fixed weights

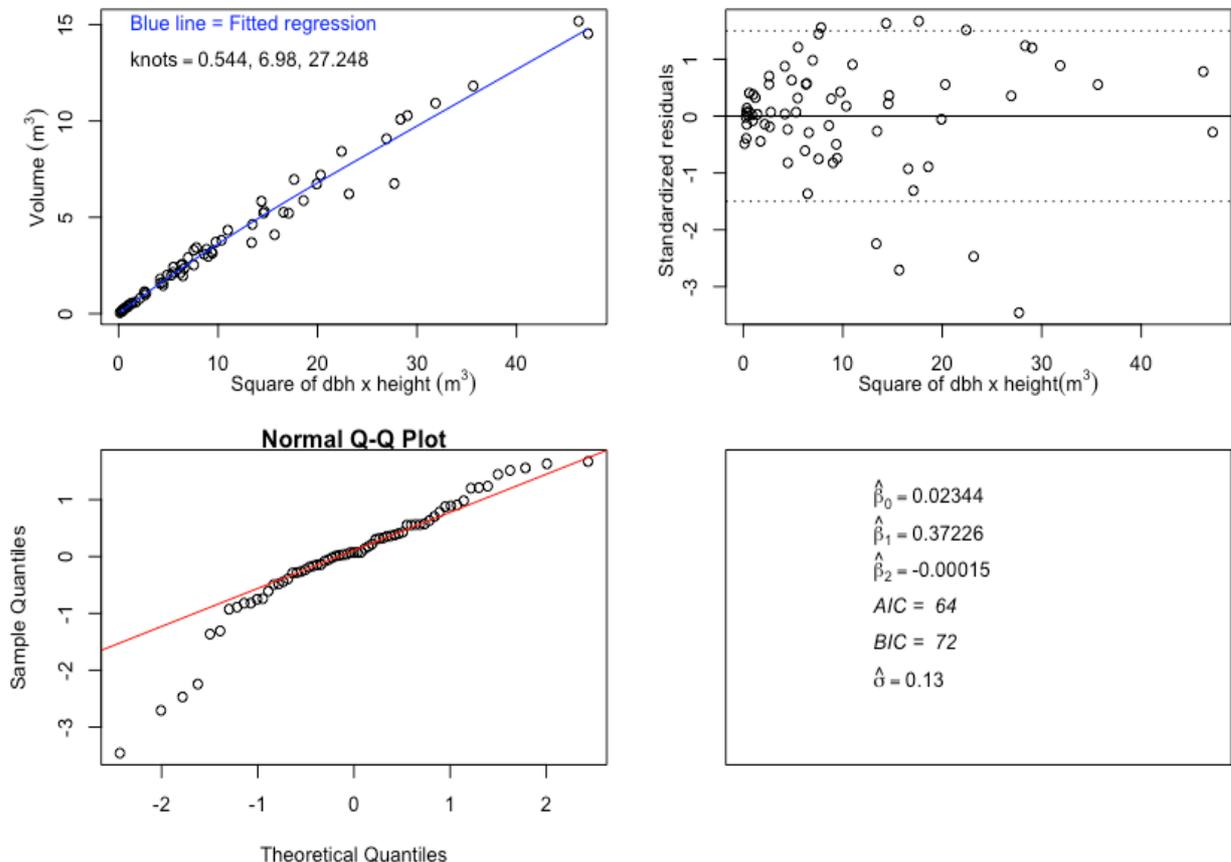
Formula: ~DBH2H.m3

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0234367	0.023417288	1.00083	0.3207
DBH2H.m3	0.3722639	0.010728671	34.69805	0.0000
DBH2H.m3.splinepoints	-0.0001530	0.000041219	-3.71088	0.0004

Plot of Model 10

T_dumosa:Model 10: (Volume ~ dbh²*H), Cubic Spline with varFixed



7.11 Model 11– Volume with square of diameter at breast height * height (DBH2H) as predictor, with varPower

```
> td.m11 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,
  na.action=na.omit, weights = varPower(form =
  ~DBH2H.m3))
> summary(td.m11)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints

Data: NULL

AIC	BIC	logLik
27.90799	38.70241	-8.953995

Variance function:

Structure: Power of variance covariate

Formula: ~DBH2H.m3

Parameter estimates:

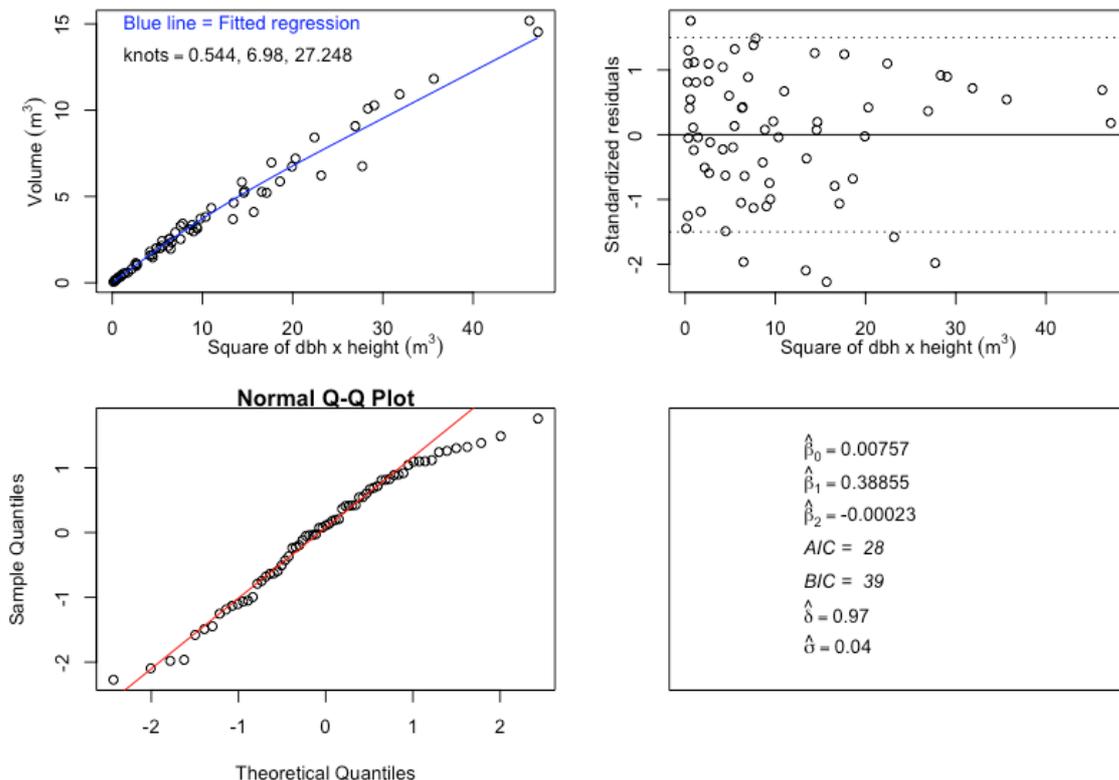
power
0.9749772

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0075704	0.004708151	1.60792	0.1128
DBH2H.m3	0.3885533	0.007899858	49.18484	0.0000
DBH2H.m3.splinepoints	-0.0002254	0.000046442	-4.85352	0.0000

Plot of Model 11

T_dumosa:Model 11: (Volume ~ dbh²*H), Cubic Spline with varPower



7.12 Model 12 –Volume with square of diameter at breast height * height (DBH2H) as predictor, with varConstPower

```
> td.m12 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,
  na.action=na.omit, weights = varConstPower(form =
  ~DBH2H.m3))
> summary(td.m12)
```

Generalized least squares fit by REML

```
Model: Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints
Data: NULL
      AIC      BIC    logLik
27.55712 40.51042 -7.778561
```

Variance function:

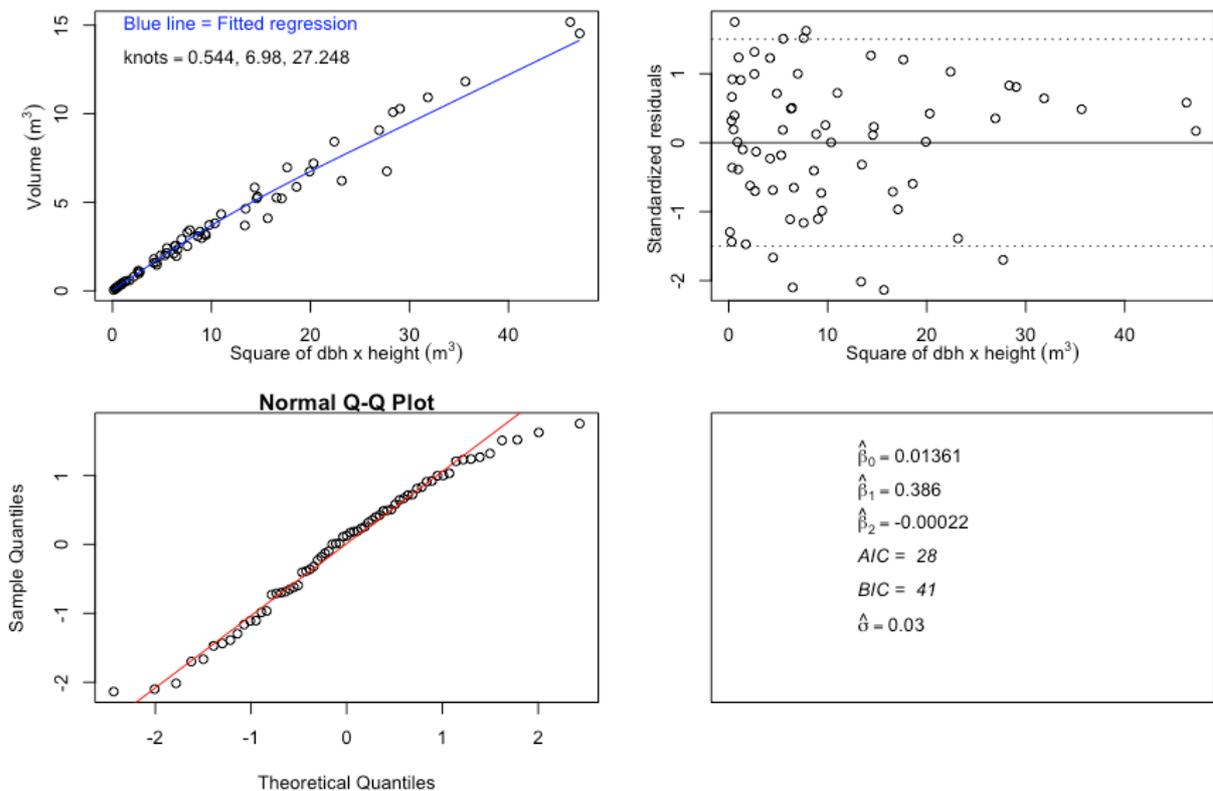
```
Structure: Constant plus power of variance covariate
Formula: ~DBH2H.m3
Parameter estimates:
      const      power
0.2998665 1.1454382
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0136115	0.006206861	2.19298	0.0319
DBH2H.m3	0.3860046	0.007614327	50.69452	0.0000
DBH2H.m3.splinepoints	-0.0002224	0.000050397	-4.41207	0.0000

Plot of Model 12

T_dumosa:Model 12: (Volume ~ dbh²*H), Cubic Spline with varConstPower



7.13 Model 13 – Volume with basal area * height (BAH) as predictor

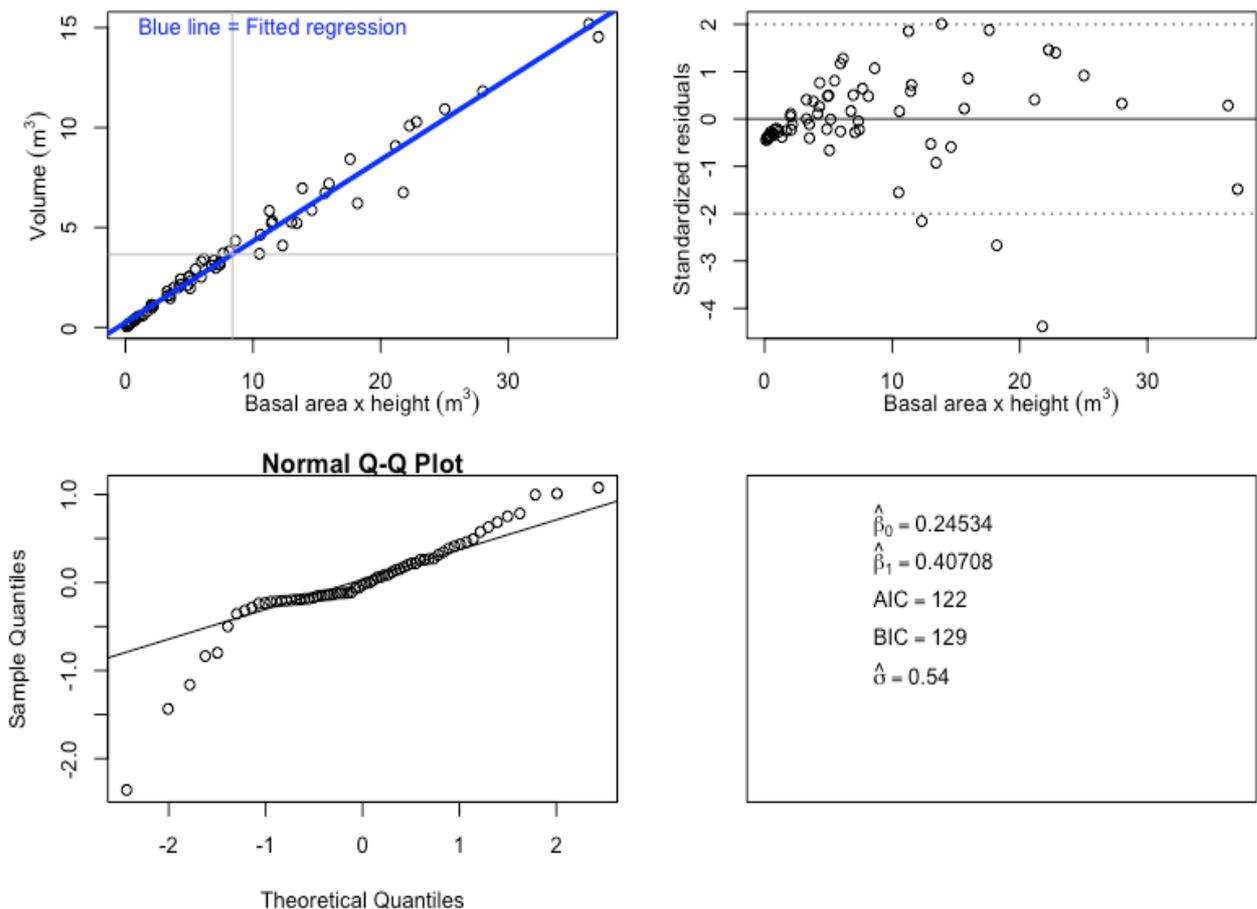
```
> td.m13 <- gls(Volume.m3 ~ BAH.m3)
> summary(td.m13)
```

Generalized least squares fit by REML

```
Model: Volume.m3 ~ BAH.m3
Data: NULL
      AIC      BIC    logLik
122.4875 129.0106 -58.24373
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.2453401	0.09218293	2.66145	0.0098
BAH.m3	0.4070815	0.00771831	52.74232	0.0000

Plot of Model 13**T_dumosa:Model 13: (Volume ~ BAH)**

7.14 Model 14 – Volume with basal area * height (BAH) as predictor, with varFixed

```
> td.m14 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
  na.action=na.omit, weights = varFixed(~BAH.m3))
> summary(td.m14)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints

Data: NULL

	AIC	BIC	logLik
	61.79515	70.43068	-26.89758

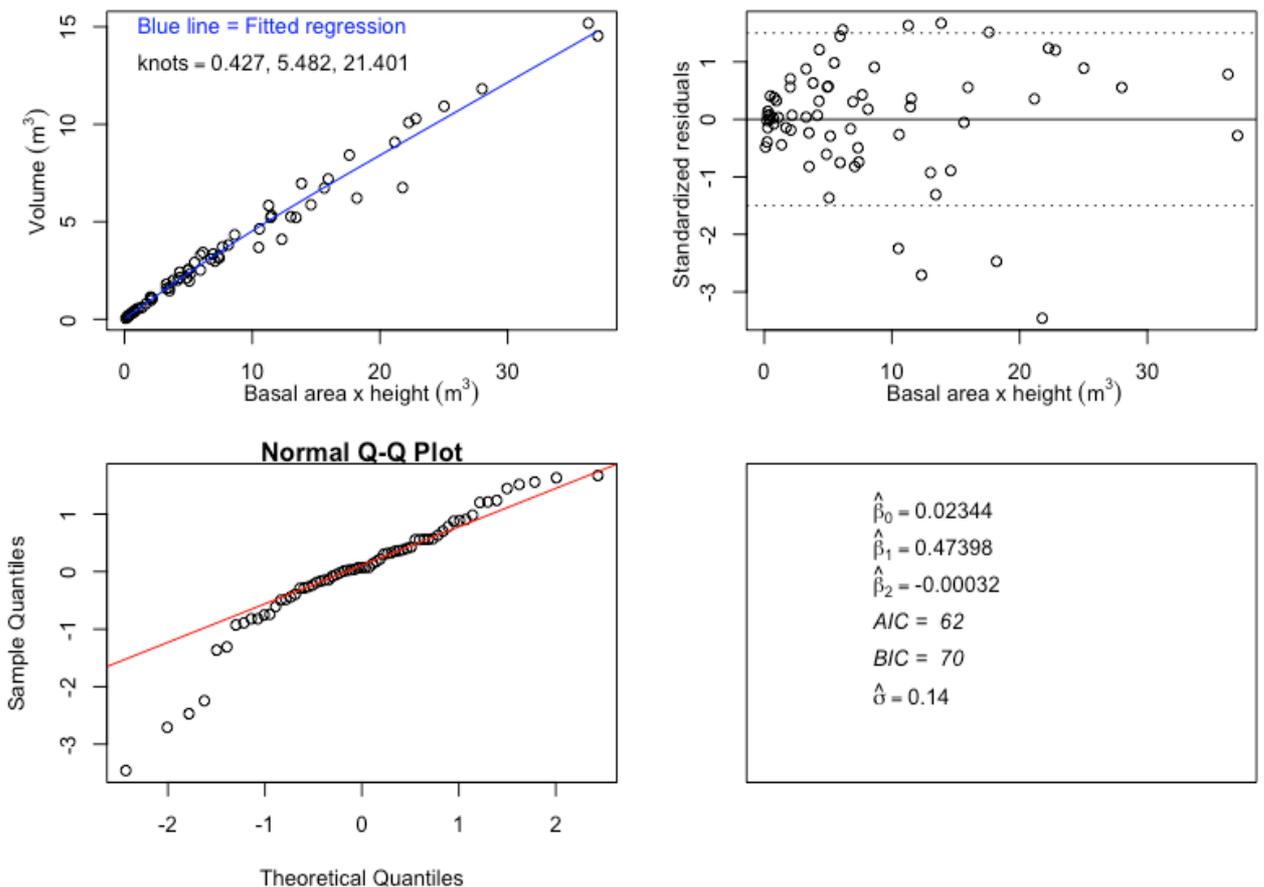
Variance function:

Structure: fixed weights

Formula: ~BAH.m3

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0234367	0.02341729	1.00083	0.3207
BAH.m3	0.4739812	0.01366017	34.69805	0.0000
BAH.m3.splinepoints	-0.0003157	0.00008508	-3.71088	0.0004

Plot of Model 14**T_dumosa:Model 14: (Volume ~ BAH), Cubic spline with varFixed**

7.15 Model 15– Volume with basal area * height (BAH) as predictor, with varPower

```
> td.m15 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
  na.action=na.omit, weights = varPower(form =
  ~BAH.m3))
> summary(td.m15)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints

Data: NULL

	AIC	BIC	logLik
	25.97547	36.76989	-7.987737

Variance function:

Structure: Power of variance covariate

Formula: ~BAH.m3

Parameter estimates:

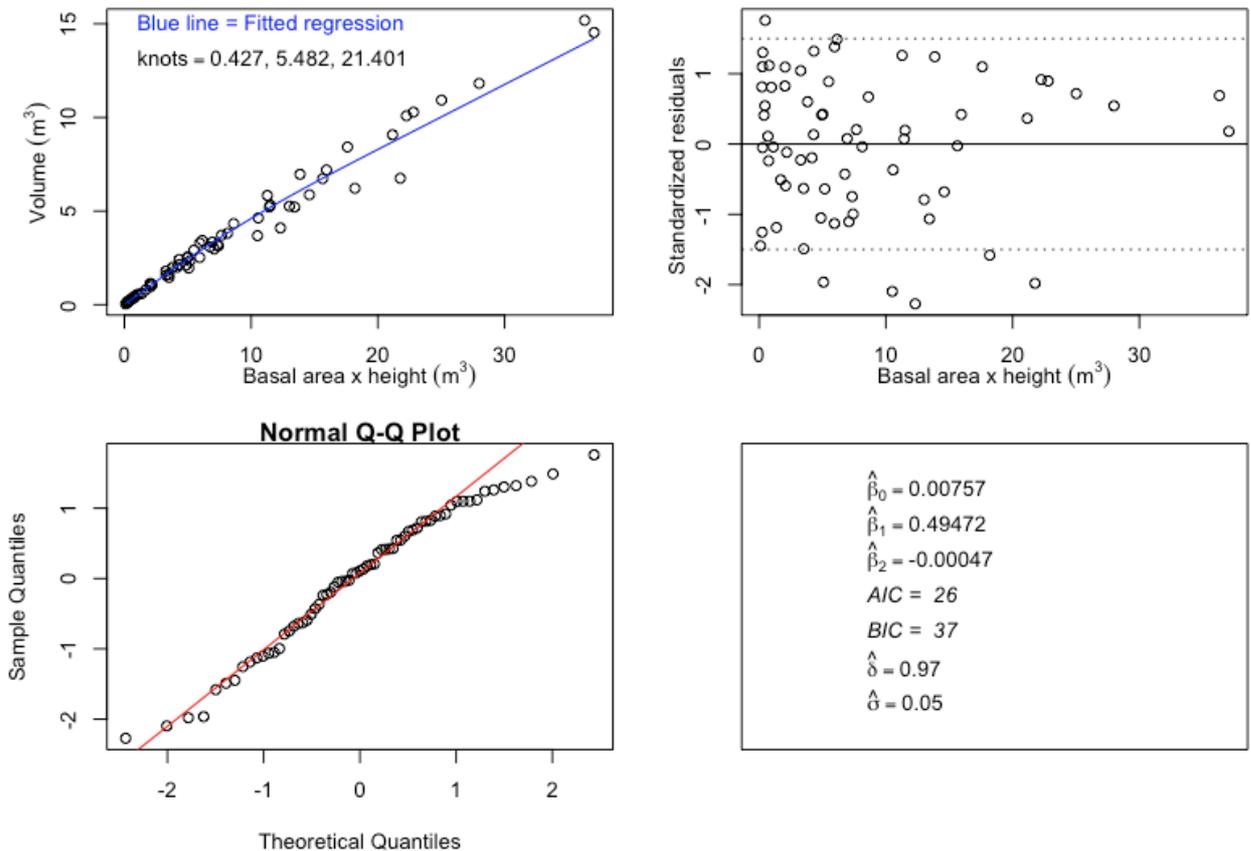
power
0.9749772

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0075704	0.004708151	1.60792	0.1128
BAH.m3	0.4947214	0.010058411	49.18484	0.0000
BAH.m3.splinepoints	-0.0004653	0.000095861	-4.85352	0.0000

Plot of Model 15

T_dumosa:Model 15: (Volume ~ BAH), Cubic spline with varPower



7.16 Model 16 – Volume with basal area * height (BAH) as predictor, with varConstPower

```
> td.m16 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
  na.action=na.omit, weights = varConstPower(form =
  ~BAH.m3))
> summary(td.m16)
```

Generalized least squares fit by REML

```
Model: Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints
Data: NULL
      AIC      BIC    logLik
25.62461 38.5779 -6.812303
```

Variance function:

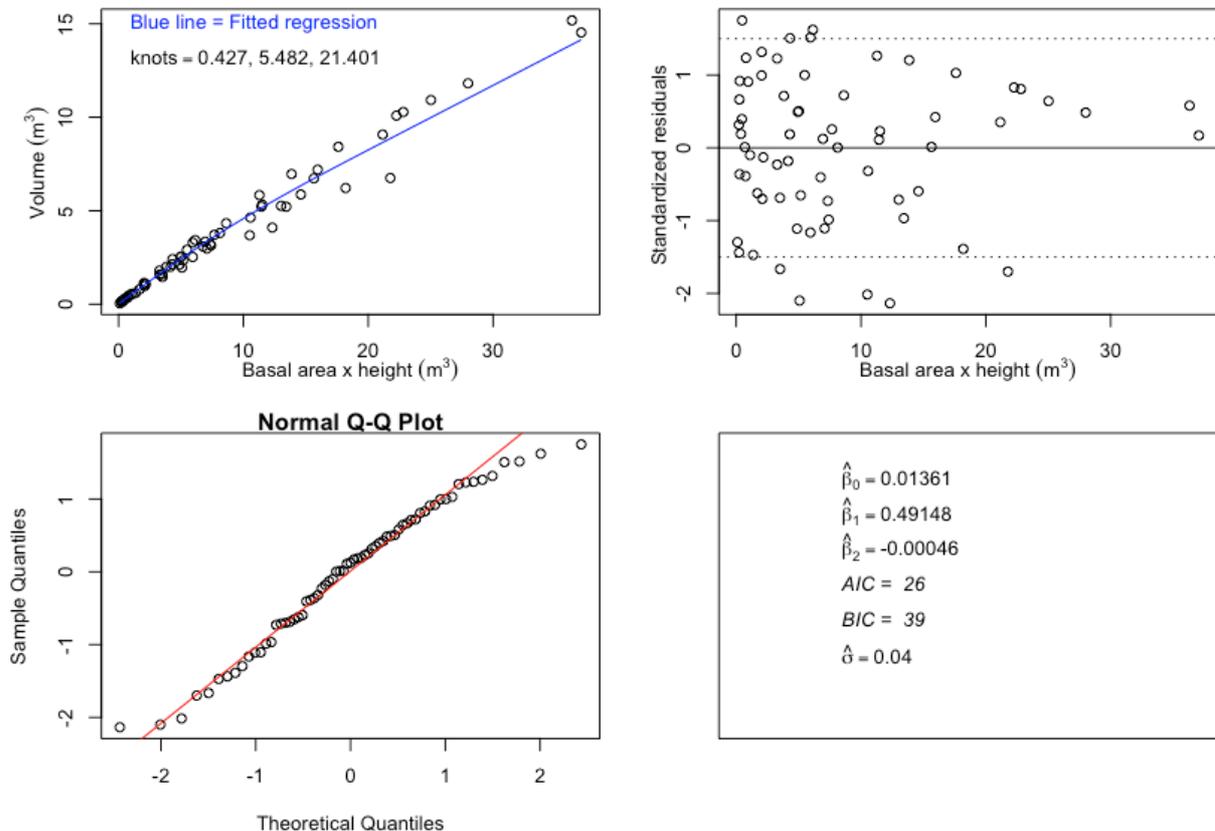
```
Structure: Constant plus power of variance covariate
Formula: ~BAH.m3
Parameter estimates:
      const      power
0.2273839 1.1454381
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0136115	0.006206860	2.19298	0.0319
BAH.m3	0.4914764	0.009694862	50.69452	0.0000
BAH.m3.splinepoints	-0.0004590	0.000104025	-4.41207	0.0000

Plot of Model 16

T_dumosa:Model 16: (Volume ~ BAH), Cubic spline with varConstPower



1. Model evaluation using AIC and BIC values

SN	Model	AIC	BIC
1	Model 1 > ad.m1 <- gls(Volume.m3 ~ DBH.cm)	240	247
2	Model 2 > ad.m2 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints, na.action=na.omit, weights = varFixed(~DBH.cm))	221	230
3	Model 3 > ad.m3 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints, na.action=na.omit, weights = varPower(form = ~DBH.cm))	167	178
4	Model 4 > ad.m4 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints, na.action=na.omit, weights = varConstPower(form = ~DBH.cm))	169	182
5	Model 5 > td.m5 <- gls(Volume.m3 ~ BA.m2)	233	240
6	Model 6 > td.m6<- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights = varFixed(~BA.m2))	153	161
7	Model 7 > td.m7 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights = varPower(form = ~BA.m2))	116	126
8	Model 8 > td.m8 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights = varConstPower(form = ~BA.m2))	118	131
9	Model 9 > td.m9 <- gls(Volume.m3 ~ DBH2H.m3)	123	129
10	Model 10 > td.m10 <-gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints, na.action=na.omit, weights = varFixed(~DBH2H.m3))	64	72

11	Model 11 <pre>> td.m11 <-gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints, na.action=na.omit, weights = varPower(form = ~DBH2H.m3))</pre>	28	39
12	Model 12 <pre>> td.m12 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints, na.action=na.omit, weights = varConstPower(form = ~DBH2H.m3))</pre>	28	41
13	Model 13 <pre>> td.m13 <- gls(Volume.m3 ~ BAH.m3)</pre>	122	129
14	Model 14 <pre>> td.m14 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints, na.action=na.omit, weights = varFixed(~BAH.m3))</pre>	62	70
15	Model 15 <pre>> td.m15 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints, na.action=na.omit, weights = varPower(form = ~BAH.m3))</pre>	26	37
16	Model 16 <pre>> td.m16 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints, na.action=na.omit, weights = varConstPower(form = ~BAH.m3))</pre>	26	39

8. Selected Models

The best fitting models have been selected based on Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) values of the fitted models. The BIC value was mainly relied upon as it imposes a stronger penalty for the number of parameters in the model that need to be estimated. Smaller the values of AIC and BIC, better the fit of the model. Therefore, for *Tsuga dumosa*, the selected models are;

1. Model 7 (Model which doesn't use height)


```
td.m7 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
              na.action=na.omit, weights = varPower(form = ~BA.m2))
```
2. Model 15 (Model which uses the height)


```
td.m15 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
               na.action=na.omit, weights = varPower(form = ~BAH.m3))
```

Two models have been selected for *Tsuga dumosa*, one without height ($X_1 = BA$ which is model 7) and one with the height ($X_1 = BAH$, which is Model 15) as predictor or explanatory variable. Both the models have been fitted with natural (restricted) cubic spline function within a linear model framework. Although, nonlinear models are more flexible, they are more complicated than the linear models. The complications involved and amount of time and efforts spent on fitting nonlinear models often fail to justify by the improvements in the models. Moreover, the models fitted with natural (restricted) cubic spline functions perform well and track the curvilinearity better than nonlinear functions that were examined.

9. Demonstration of use of the selected best fit model

In general, the natural spline predictor with knots represented by t_1 , t_2 and t_3 takes the following form;

$$Y = \beta_0 + \beta_1 X + \beta_2 X_s + \varepsilon \quad (7)$$

Where X_s corresponds to value in X as follows:

$$X_s = g(X) = (X - t_1)_+^3 - (X - t_2)_+^3 \frac{(t_3 - t_1)}{(t_3 - t_2)} + (X - t_3)_+^3 \frac{(t_2 - t_1)}{(t_3 - t_2)} \quad (8)$$

and the value of the positive part functions depend on the values of the knots as follows;

$$(X - t_1)_+^3 = (X - t_1)_+^3, \text{ if } X > t_1 \text{ and } (X - t_1)_+^3 = 0, \text{ if } X < t_1 \quad (9)$$

$$(X - t_2)_+^3 = (X - t_2)_+^3, \text{ if } X > t_2, \text{ and } (X - t_2)_+^3 = 0, \text{ if } X < t_2 \quad (10)$$

$$(X - t_3)_+^3 = (X - t_3)_+^3, \text{ if } X > t_3, \text{ and } (X - t_3)_+^3 = 0, \text{ if } X < t_3 \quad (11)$$

Where t_1 , t_2 and t_3 for the above models are 10th, 50th and 90th percentiles and are called knots. The values of knots differ from species and models.

To demonstrate use of the selected models for *Tsuga dumosa* – model 7, the knots t_1 , t_2 and t_3 are 0.031, 0.208 and 0.613 as generated by the model. The model 7 has been fitted with volume as function of basal area in meter square (BA) i.e

$$BA = \pi r^2 \quad (12)$$

where in

$$r^2 = \left[\frac{dbh}{2 \times 100} \right]^2 \quad (13)$$

Where r is radius in meters and dbh is diameter at breast height in centimeters.

Therefore, *Tsuga dumosa* with diameter of 67.3 cm resulting in basal area of 0.355729605 m², the volume can be estimated using the above equation (model 7) as below. But first the value of BA.m² has to be calculated, which is;

$$\begin{aligned} BA &= \pi r^2 = \frac{\pi * 67.3^2}{200^2} = 0.355729605 \text{ m}^2 \\ g(X) &= (X - t_1)_+^3 - (X - t_2)_+^3 \frac{(t_3 - t_1)}{(t_3 - t_2)} + (X - t_3)_+^3 \frac{(t_2 - t_1)}{(t_3 - t_2)} \\ g(BA) &= (BA - t_1)_+^3 - (BA - t_2)_+^3 \frac{(t_3 - t_1)}{(t_3 - t_2)} + (BA - t_3)_+^3 \frac{(t_2 - t_1)}{(t_3 - t_2)} \\ g(BA) &= (0.355729605 - 0.031)_+^3 - (0.355729605 - 0.208)_+^3 \frac{(0.613 - 0.031)}{(0.613 - 0.208)} + 0 \\ &= (0.324729605)_+^3 - (0.147729605)_+^3 \frac{(0.582)}{(0.405)} + 0 \\ &= (0.324729605)_+^3 - (0.147729605)_+^3 * 1.4370370 + 0 \\ &= 0.034242515 - 0.003224056 * 1.4370370 \\ &= 0.034242515 - 0.004633088 \\ &= 0.029609 \end{aligned}$$

Hence, the volume predicted for this tree by the selected model (model 7) is

$$\begin{aligned} V &= \beta_0 + \beta_1 \cdot BA + \beta_2 BA \cdot m_2 + \varepsilon \\ &= -0.154786 + 14.475407 * 0.355729605 + (-6.321858 * 0.029609) \\ &= -0.154786 + 5.149331 - 0.187184 \\ &= \mathbf{4.807358 \text{ m}^3} \end{aligned}$$

Similarly, to demonstrate model 15 with t_1 , t_2 and t_3 of 0.427, 5.482 and 21.401 respectively, we considered this same tree but with height, i.e $dbh = 67.3$ cm resulting in $BA = 0.355729605$ m² and height (H) = 36.6 m.

$$\begin{aligned} BAH &= 0.355729605 \times 36.6 \\ &= 13.01970353 \end{aligned}$$

$$\begin{aligned} g(X) &= (X - t_1)_+^3 - (X - t_2)_+^3 \frac{(t_3 - t_1)}{(t_3 - t_2)} + (X - t_3)_+^3 \frac{(t_2 - t_1)}{(t_3 - t_2)} \\ g(BAH) &= (BAH - t_1)_+^3 - (BAH - t_2)_+^3 \frac{(t_3 - t_1)}{(t_3 - t_2)} + (BAH - t_3)_+^3 \frac{(t_2 - t_1)}{(t_3 - t_2)} \end{aligned}$$

$$\begin{aligned}
&= (13.01970353 - 0.427)_+^3 - (13.01970353 - 5.482)_+^3 \frac{(21.401 - 0.427)}{(21.401 - 5.482)} + 0 \\
&= (12.59270353)_+^3 - (7.53770353)_+^3 \frac{(20.974)}{(15.919)} + 0 \\
&= 1996.902849 - 428.269509 * 1.317545 + 0 \\
&= 11996.902849 - 564.264381 + 0 \\
&= 1432.638468
\end{aligned}$$

Hence, the volume predicted by model 15 for this tree is;

$$\begin{aligned}
V &= \beta_0 + \beta_1 \cdot BAH \cdot m^3 + \beta_2 BAH \cdot m^3_2 + \varepsilon \\
&= 0.0075704 + 0.4947214 * 13.01970353 + (-0.0004653 * 1432.638468) \\
&= 0.0075704 + 6.441126 + (-0.666607) \\
&= \mathbf{5.782089 \text{ m}^3}
\end{aligned}$$

However, the field measured volume for this particular tree with DBH of 67.3 cm and height of 36.6 m is 5.261418566 m³.

10. Model Performance

To assess the performance of selected models, we compared the volume predicted by selected models (7 and 15) with the volume of the tree as measured in the field. Using the equations of the selected models, volume prediction or estimation was done in R.

SN	Tree_ID	Height (in m)	DBH (in cm)	Volume in m ³ (Field measured) [A]	Predicted Volume Model_7 [B]	Predicted Volume Model_15 [C]	Difference (Field - Model_7) [A - B]	Difference (Field - Model_15) [A - C]
1	tde01	36.6	67.3	5.261418566	4.807358221	5.78208968	0.454060346	-0.520671114
2	tde02	31.02	38	1.460434861	1.483353136	1.734270741	-0.022918275	-0.273835881
3	tde03	35.3	81	6.215342326	6.849915937	7.65687807	-0.634573612	-1.441535744
4	tde04	36.55	45.5	2.523334832	2.184465223	2.869632725	0.338869608	-0.346297893
5	tde05	29.52	54	3.086679845	3.111397397	3.235305524	-0.024717552	-0.148625679
6	tde06	14.1	26	0.368229579	0.613687394	0.377908365	-0.245457815	-0.009678786
7	tde07	11.3	15.7	0.125762664	0.125447448	0.115795582	0.000315216	0.009967082
8	tdec01	29.55	56.5	3.140540624	3.408120574	3.518864741	-0.267579951	-0.378324117
9	tdec02	23	34	0.972963801	1.158114182	1.038522755	-0.185150381	-0.065558954
10	tdec03	35	71	6.964498285	5.32938342	6.096017286	1.635114865	0.868480999
11	tdec04	27.3	48	2.514826117	2.443304552	2.40876784	0.071521565	0.106058277
12	tdec05	28.55	47.3	2.554924699	2.369619533	2.444453136	0.185305167	0.110471563
13	tdec06	27	45	2.134944867	2.134156707	2.105075621	0.000788161	0.029869246
14	tdec07	24.6	24	0.555528384	0.500047734	0.557984722	0.05548065	-0.002456338
15	tdec08	25.2	22	0.523168201	0.39546939	0.481411424	0.127698811	0.041756777
16	tdec09	32.7	75.4	5.870714905	5.97839663	6.370865646	-0.107681725	-0.500150741
17	tdec10	36	86.5	9.075542558	7.758451829	8.689968605	1.317090729	0.385573953
18	tdec11	33.7	63	3.686736391	4.227834049	4.806086247	-0.541097658	-1.119349856
19	tdec12	26.3	28.7	0.802280421	0.781421915	0.848332114	0.020858506	-0.046051694
20	tdec13	34.3	52.2	3.20821062	2.904326361	3.489249358	0.303884259	-0.281038738
21	tdec14	32.3	58.3	4.331624883	3.628217284	4.036134265	0.703407599	0.295490618
22	tdec15	31.3	57.5	3.811423087	3.529735514	3.827405866	0.281687573	-0.015982779
23	tdec16	25.3	42	1.612550626	1.84283488	1.728081642	-0.230284254	-0.115531015
24	tdec17	29.4	46.9	1.957068704	2.327932869	2.473442032	-0.370864166	-0.516373329
25	tdec18	21.6	35.8	1.067261767	1.30016821	1.080736933	-0.232906442	-0.013475165
26	tdec19	19.45	22.7	0.444718405	0.43103967	0.396971932	0.013678735	0.047746473
27	tdec20	29.7	37.5	1.580749126	1.440803644	1.61957911	0.139945482	-0.038829984
28	tdec21	35.6	74.8	6.730652309	5.888071747	6.750730738	0.842580562	-0.02007843
29	tdec22	36.1	75	7.197809115	5.918115564	6.860607434	1.279693551	0.337201681
30	tdec23	35.8	61.3	4.633258308	4.006834621	4.830219463	0.626423687	-0.196961154
31	tdec28	15.2	20	0.289943898	0.299972322	0.243810734	-0.010028424	0.046133165
32	tdec29	14.6	18.3	0.206236635	0.225949037	0.197549407	-0.019712402	0.008687228
33	tdec30	13.6	16.3	0.168537707	0.147275847	0.14796962	0.02126186	0.020568086

34	tdec31	13.9	14.7	0.107751191	0.090885815	0.124278244	0.016865376	-0.016527054
35	tdec32	40.6	64.9	5.21467086	4.480291092	5.936848925	0.734379769	-0.722178065
36	tdw01	23.2	33.5	1.135850828	1.119915827	1.017245014	0.015935001	0.118605813
37	tdw02	22.9	27.5	0.593736665	0.704846709	0.680093727	-0.111110044	-0.086357062
38	tdw03	31.6	47	2.911417256	2.338325789	2.659725046	0.573091467	0.251692211
39	tdw04	8.6	19.5	0.150392949	0.277518631	0.134633151	-0.127125682	0.015759799
40	tdw05	35.3	95	10.92360789	9.275715883	10.03034944	1.647892005	0.89325845
41	tdw07	30.6	56.5	3.71350107	3.408120574	3.632560151	0.305380496	0.080940919
42	tdw08	30.6	69	5.218612075	5.044527945	5.176187844	0.17408413	0.042424231
43	tdwc01	30.5	37	1.798330846	1.39879175	1.619156771	0.399539096	0.179174075
44	tdwc02	17.9	18	0.24656114	0.213568242	0.232915749	0.032992898	0.013645391
45	tdwc03	44.5	89.5	11.81685746	8.277758955	11.06170765	3.539098505	0.755149809
46	tdwc04	46.4	69.5	8.420746233	5.115150379	7.449988296	3.305595854	0.970757937
47	tdwc05	34.8	47.4	3.429788227	2.380088994	2.95893862	1.049699233	0.470849607
48	tdwc06	42.7	58	5.835085848	3.591163155	5.113360903	2.243922692	0.721724944
49	tdwc07	29.8	29.5	1.102403791	0.834268201	1.013281969	0.268135591	0.089121823
50	tdw09	25.14	105	6.750827758	11.24261793	8.90258126	-4.491790176	-2.151753502
51	tdw10	7.5	21.5	0.14149372	0.370743143	0.142276965	-0.229249423	-0.000783245
52	tdw11	8	12.7	0.049332429	0.028583925	0.057706207	0.020748505	-0.008373777
53	tdw12	20	51.5	2.002121656	2.825306866	2.044326438	-0.82318521	-0.042204782
54	tdw13	23	62	3.343592775	4.097273332	3.315982791	-0.753680557	0.027609984
55	tdw14	21	56	2.346031083	3.347936557	2.51670711	-1.001905473	-0.170676027
56	tdw15	23	46	2.000308838	2.235265534	1.880367865	-0.234956696	0.119940974
57	tdw16	22	50	2.413907763	2.658874078	2.117167218	-0.244966315	0.296740544
58	tdw17	32	70	4.097460357	5.186166916	5.51392997	-1.088706559	-1.416469613
59	tdw18	28	52	3.29507991	2.881662641	2.871209745	0.413417269	0.423870164
60	tdw19	39	110	14.53471499	12.29982779	14.20542544	2.234887208	0.329289555
61	tdw20	11.5	27.7	0.354556887	0.71738437	0.350414643	-0.362827483	0.004142244
62	tdw21	20.7	117	10.08938058	13.86253147	9.071248149	-3.773150882	1.018132435
63	tdw22	25	136	15.18178732	18.58998055	13.94672554	-3.408193226	1.235061781
64	tdw23	24	110	10.2817002	12.29982779	9.262895759	-2.01812759	1.018804438
65	tdw24	17.3	72.2	2.975348688	5.503330277	3.376942917	-2.527981589	-0.401594228
66	tdw25	19.1	87.6	5.317951188	7.946834496	5.203236886	-2.628883308	0.114714302
67	tdw26	16.4	61.5	2.111685948	4.032593892	2.37686616	-1.920907944	-0.265180211
				244.9845135	248.2289023	244.8180019	-3.244388788	0.166511619

From the above table, the difference [A-B] provides difference between the volume measured in the field (actual volume) and the volume predicted by model 7. The figures with negative (-) indicates that the volume has been over-predicted by the model 7 vis-à-vis actual volume of the particular tree. And the figures without negative (-) sign indicates the under prediction of volume by the model 7.

Similarly, the difference [A-C] is the difference between the actual volume and the volume predicted by the model 15. Same explanation is applicable here – the figures with negative sign

indicates overprediction of volume by the model and vice-versa, while those figures without (-) are under prediction of volume by the model 15.

Summation of the figures in the difference column result in -3.244388788 and 0.166511619 for model 7 and model 15 respectively. These indicate that the model 7 over predicts total volume for 67 trees by 3.244388788 m³, while the model 15 under predicts the total volume of 67 trees by 0.166511619 m³.

11. Limitations of the model

The model has the following limitations;

1. The modeling has been done based on only 67 sample trees. The model can be further improved by increasing the samples.
2. The diameter for the samples ranges between minimum of 12.7 cm to 136 cm (over bark). However, the model prediction for trees with bigger diameters should be done with caution since there were limited sample trees from bigger diameter class.

12. Conclusion

Both the models - model 7 which doesn't use the height as predictor as well as the model 15 with height as predictor perform very well, as empirically shown above. Like other conifer species that we have modelled (*Pinus wallichiana*, *Juniperus recurva*) for which the model with height as predictor was found to be a better for *Tsuga dumosa*.

This, therefore, it leads us to conclude that the best model for *Tsuga dumosa*, out of 16 models fitted above, is model 15, since it has the lowest AIC and BIC values of 26 and 37 respectively, as compared to model 7 which has 116 and 126 as the values for AIC and BIC respectively.

Since the models have been fitted with different predictors (one with and other without height as predictor), it leads us to consider two best fit models for *Tsuga dumosa*, namely;

1. Model 7: the best fit model that doesn't use height
2. Model 15: the best fit model which uses height as predictor.

13. Acknowledgement

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15. Annexure – Dataset for *Tsuga dumosa*

SN	Tree_ID	Height.m	DBH.cm	Volume.m3	BA.m2	BAH.m3	DBH2H.m3
1	tde01	36.6	67.3	5.261419	0.35573	13.0197	16.5772
2	tde02	31.02	38	1.460435	0.113411	3.518025	4.479288
3	tde03	35.3	81	6.215342	0.5153	18.19008	23.16033
4	tde04	36.55	45.5	2.523335	0.162597	5.942922	7.566764
5	tde05	29.52	54	3.08668	0.229022	6.760733	8.608032
6	tde06	14.1	26	0.36823	0.053093	0.74861	0.95316
7	tde07	11.3	15.7	0.125763	0.019359	0.21876	0.278534
8	tdec01	29.55	56.5	3.140541	0.250719	7.408738	9.433099
9	tdec02	23	34	0.972964	0.090792	2.088217	2.6588
10	tdec03	35	71	6.964498	0.395919	13.85717	17.6435
11	tdec04	27.3	48	2.514826	0.180956	4.940092	6.28992
12	tdec05	28.55	47.3	2.554925	0.175716	5.016702	6.387463
13	tdec06	27	45	2.134945	0.159043	4.294164	5.4675
14	tdec07	24.6	24	0.555528	0.045239	1.112878	1.41696
15	tdec08	25.2	22	0.523168	0.038013	0.957934	1.21968
16	tdec09	32.7	75.4	5.870715	0.446511	14.60092	18.59047
17	tdec10	36	86.5	9.075543	0.587655	21.15556	26.9361
18	tdec11	33.7	63	3.686736	0.311725	10.50512	13.37553
19	tdec12	26.3	28.7	0.80228	0.064692	1.701412	2.166305
20	tdec13	34.3	52.2	3.208211	0.214008	7.340489	9.346201
21	tdec14	32.3	58.3	4.331625	0.266948	8.622427	10.97841
22	tdec15	31.3	57.5	3.811423	0.259672	8.127742	10.34856
23	tdec16	25.3	42	1.612551	0.138544	3.505169	4.46292
24	tdec17	29.4	46.9	1.957069	0.172757	5.079055	6.466853
25	tdec18	21.6	35.8	1.067262	0.10066	2.174251	2.768342
26	tdec19	19.45	22.7	0.444718	0.040471	0.787157	1.002239
27	tdec20	29.7	37.5	1.580749	0.110447	3.280265	4.176563
28	tdec21	35.6	74.8	6.730652	0.439433	15.64383	19.91834
29	tdec22	36.1	75	7.197809	0.441786	15.94849	20.30625
30	tdec23	35.8	61.3	4.633258	0.295128	10.56559	13.45253
31	tdec28	15.2	20	0.289944	0.031416	0.477522	0.608
32	tdec29	14.6	18.3	0.206237	0.026302	0.384012	0.488939
33	tdec30	13.6	16.3	0.168538	0.020867	0.283795	0.361338
34	tdec31	13.9	14.7	0.107751	0.016972	0.235906	0.300365
35	tdec32	40.6	64.9	5.214671	0.33081	13.43091	17.10076
36	tdw01	23.2	33.5	1.135851	0.088141	2.044878	2.60362
37	tdw02	22.9	27.5	0.593737	0.059396	1.360162	1.731813
38	tdw03	31.6	47	2.911417	0.173494	5.482425	6.98044
39	tdw04	8.6	19.5	0.150393	0.029865	0.256837	0.327015

40	tdw05	35.3	95	10.92361	0.708822	25.02141	31.85825
41	tdw07	30.6	56.5	3.713501	0.250719	7.671993	9.768285
42	tdw08	30.6	69	5.218612	0.373928	11.4422	14.56866
43	tdwc01	30.5	37	1.798331	0.107521	3.279391	4.17545
44	tdwc02	17.9	18	0.246561	0.025447	0.4555	0.57996
45	tdwc03	44.5	89.5	11.81686	0.629124	27.996	35.64561
46	tdwc04	46.4	69.5	8.420746	0.379367	17.60263	22.41236
47	tdwc05	34.8	47.4	3.429788	0.17646	6.140812	7.818725
48	tdwc06	42.7	58	5.835086	0.264208	11.28168	14.36428
49	tdwc07	29.8	29.5	1.102404	0.068349	2.036808	2.593345
50	tdw09	25.14	105	6.750828	0.865901	21.76876	27.71685
51	tdw10	7.5	21.5	0.141494	0.036305	0.272288	0.346688
52	tdw11	8	12.7	0.049332	0.012668	0.101341	0.129032
53	tdw12	20	51.5	2.002122	0.208307	4.166145	5.3045
54	tdw13	23	62	3.343593	0.301907	6.943862	8.8412
55	tdw14	21	56	2.346031	0.246301	5.172318	6.5856
56	tdw15	23	46	2.000309	0.16619	3.822376	4.8668
57	tdw16	22	50	2.413908	0.19635	4.31969	5.5
58	tdw17	32	70	4.09746	0.384845	12.31504	15.68
59	tdw18	28	52	3.29508	0.212372	5.946407	7.5712
60	tdw19	39	110	14.53471	0.950332	37.06294	47.19
61	tdw20	11.5	27.7	0.354557	0.060263	0.693022	0.882384
62	tdw21	20.7	117	10.08938	1.075132	22.25522	28.33623
63	tdw22	25	136	15.18179	1.452672	36.31681	46.24
64	tdw23	24	110	10.2817	0.950332	22.80796	29.04
65	tdw24	17.3	72.2	2.975349	0.409415	7.082888	9.018213
66	tdw25	19.1	87.6	5.317951	0.602696	11.51149	14.65688
67	tdw26	16.4	61.5	2.111686	0.297057	4.871738	6.20289