

དབལ་ཕྱན་འབྲུག་གཞུང་། རོ་ནམ་དང་ནགས་
ཚལ་ལྷན་ཁག། རགས་ཚལ་དང་སྤོང་ཀ་འབྲས་
རྟོག་ལས་ཁུངས།



Royal Government of Bhutan
Ministry of Agriculture and Forests
Department of Forests and Park Services

SPECIES SPECIFIC VOLUME EQUATION TO ESTIMATE MERCHANTABLE VOLUME

Quercus lanata

Forest Resources Management Division
Department of Forest and Park Services
Ministry of Agriculture and Forests

2018

Species specific volume equation to
estimate merchantable volume

Quercus lanata

December, 2018

Table of Contents

1.	<i>Summary</i>	1
2.	<i>Introduction</i>	2
3.	<i>Volume Calculation</i>	3
4.	<i>The Dataset used for modeling volume of Quercus lanata</i>	4
4.1	Summary descriptive statistics of <i>Quercus lanata</i> dataset.....	4
5.	<i>Fitting the models</i>	5
6.	<i>Summary Plots</i>	6
7.	<i>Models and results</i>	7
7.1	Model 1 - Volume with diameter at breast height (DBH) as predictor	7
7.2	Model 2 - Volume with diameter at breast height (DBH) as predictor, with varFixed	8
7.3	Model 3- Volume with diameter at breast height (DBH) as predictor, with varPower.....	9
7.4	Model 4 - Volume with diameter at breast height (DBH) as predictor, with varConstPower ...	10
7.5	Model 5 - Volume with basal area (BA) as predictor.....	11
7.6	Model 6 - Volume with basal area (BA) as predictor, with varFixed.....	12
7.7	Model 7 Volume with basal area (BA) as predictor, with varPower.....	13
7.8	Model 8 – Volume with basal area (BA) as predictor, with varConstPower.....	14
7.9	Model 9 – Volume with square of diameter at breast height * height (DBH2H) as predictor..	15
7.10	Model 10 – Volume with square of diameter at breast height * height (DBH2H) as predictor, with varFixed.....	16
7.11	Model 11– Volume with square of diameter at breast height * height (DBH2H) as predictor, with varPower.....	17
7.12	Model 12 –Volume with square of diameter at breast height * height (DBH2H) as predictor, with varConstPower	18
7.13	Model 13 – Volume with basal area * height (BAH) as predictor.....	19
7.14	Model 14 – Volume with basal area * height (BAH) as predictor, with varFixed	20
7.15	Model 15– Volume with basal area * height (BAH) as predictor, with varPower.....	21
7.16	Model 16 – Volume with basal area * height (BAH) as predictor, with varConstPower	22
8.	<i>Model evaluation using AIC and BIC values</i>	23
9.	<i>Selected Models</i>	25
10.	<i>Demonstration of use of the selected best fit model</i>	25
11.	<i>Model Performance</i>	28
12.	<i>Limitations of the model</i>	29
13.	<i>Conclusion</i>	30
14.	<i>Acknowledgement</i>	31
15.	<i>References</i>	32
16.	<i>Annexure – Dataset for Quercus lanata</i>	34

1. Summary

The volume equation developed in this study will predict the merchantable volume of *Quercus lanata*. The standards of merchantability adopted for this study to develop merchantable log volume equation are trees of 10 cm and above diameter at breast height (dbh) and the sections up to 10 cm top diameter over the bark.

A total of 16 models were fitted. First 4 models were fitted with volume as a function of diameter at breast height (DBH), while models 5 – 8 were fitted with basal area (BA) as the predictor variable. With product of squared diameter at breast height and height (DBH²H) as predictor variable, 4 models, namely the models 9 – 12 were fitted. The last four models, 13 -16 were fitted with product of basal area and height (BAH) as the predictor.

The initial plots of response variable (volume) and predictor variables (DBH, BA, DBH²H and BAH) clearly indicated presence of heteroscedasticity, which has been modeled using variance functions (varFixed, varPower and varConstPower) in gls () function of nlme package.

Of the sixteen, two models viz model 7 (fitted without height as predictor) and model 15 (fitted with height as predictor) with the lowest values of AIC and BIC have been selected as the best fit models for *Quercus lanata*. The model 7 had AIC and BIC values of 21 and 30 respectively, while the model 15 had AIC and BIC values of 0 (-0.228 to be precise) and 9 respectively. Lower the AIC and BIC values, better the fit of the model.

The performance of the selected models was assessed by comparing the actual volume with the volumes predicted by two selected models for each tree. Therefore, two models – one with height (Model 15) and one without (Model 7) as predictor have been selected as best fit models for *Quercus lanata*.

2. Introduction

The volume equations, developed during pre-investment survey (PIS) carried out between 1974-81 predict total tree volume, and not the merchantable volume of trees. The recent change of policy of the Department of Forests and Park Services to allot timber for rural house construction in the form of log volume instead of allotting by number of trees as was once practiced, has necessitated development of merchantable log volume equation.

Therefore, standards of merchantability adopted for this study to develop merchantable log volume equation are trees of 10 cm and above diameter at breast height (dbh) and the sections up to 10 cm top diameter over the bark.

As was done for PIS exercise to develop volume equation, this study ignores/does not consider the volume of foliage and branches for the purpose of calculating the merchantable volume. This decision stems from the objective, which is to estimate merchantable log volume. Moreover, branches are rarely used as timber (at least in Bhutan) and are mostly used for firewood.

The sample trees for this study have been felled as part of biomass equation development field work. The data protocol for biomass equation development required collecting a minimum of 8 trees each from four regions of Bhutan namely, eastern, eastern central, western and western central. Therefore, 49 trees in total have been felled for *Quercus lanata* from four regions namely; eastern, east-central, western-central and western regions.

The trees were felled at 0.3 m height from the ground at which the diameter was measured and recorded. After felling diameter was measured at 0.7 m from 0.3 m height (essentially making 1 m height, i.e. $0.3\text{ m} + 0.7\text{ m} = 1\text{ m}$). Thereafter, at every meter length, the diameter was measured and recorded, thus making many 1 m length sections of log. As mentioned above the smallest top diameter considered for merchantable log volume calculation was up to 10 cm diameter over bark. Top sections below 10 cm diameter have been discarded.

3. Volume Calculation

Trees after felling are converted into different sizes of sections depending on the requirement and demand. Sections with length of 8 or more feet long are called logs and shorter ones are called sticks or bolts (Avery and Burkhart, 1994). The scaling or measuring the volume of the section is done by multiplying the length with the cross-sectional area of the section. Although they rarely form true circles, they are assumed so for the purpose of calculating cross sectional area in meter square, which is;

$$\text{Cross sectional area (A)} = A = \pi r^2 = \frac{\pi D^2}{4 \times 10000} \quad (1)$$

Where **r** is radius in meters and **D** is diameter at breast height in centimeters.

From the ground level to 0.3 m height (height at which sample tree has been cut) is section I, while 0.3 m to 0.7 m is section II. The subsequent sections of 1 m length each are numbered III, IV and so on. The last section is the terminal section, whose length is equal to or less than 1 m. As was adopted for PIS, in this study too the branch volumes are ignored assuming that rarely branches yield merchantable timber.

The diameter at zero height (ground level) for stump wasn't measured in the field (for those sample trees for which volume data was collected during biomass equation development field work) and therefore, calculated based on diameter reading at 0.3 m height. Therefore, diameter at zero height was calculated as 10% more than diameter at 0.3 m height, which is;

$$D_{(\text{ground})} = D_{(0.3 \text{ m})} + 10\% * D_{(0.3 \text{ m})} \quad (2)$$

Where;

$D_{(\text{ground})}$ is diameter in centimeter of tree at ground level
 $D_{(0.3 \text{ m})}$ is diameter in centimeter of tree at 0.3 m height

For instance, if $D_{(0.3 \text{ m})}$ was 70 cm, the $D_{(\text{ground})}$ is calculated as;

$$\begin{aligned} D_{(\text{ground})} &= 70 \text{ cm} + 10\% \text{ of } 70 \text{ cm} \\ &= 70 + 7 \\ &= 77 \text{ cm} \end{aligned}$$

The most commonly used formulae for calculating volume are the Huber, Newton and Smalian's formulae (Sadiq, 2006, and Goulding, 1979). Of the three commonly used volume calculation approaches or formulae, I have used Smalian's formula to calculate volume (in m³) for this study, which is;

$$\text{Section volume (V}_s\text{)} = \frac{A+a}{2} * L \quad (3)$$

Where A = Cross sectional area in m² at large end of the section
a = Cross sectional area in m² at small end of the section
L = Length of the section in meter

Smalian's formula is the easiest and least expensive to apply and therefore applied to get volume for each section of the sample trees. However, for the terminal section, the following formula was used to calculate the volume, which is;

$$\text{Terminal section volume (V}_t\text{)} = \frac{A}{3} * L \quad (4)$$

The volume for sections and terminal section for individual trees were then summed to obtain the total volume for each individual sample tree, which is;

$$\text{Volume of tree (V)} = \sum_{s=1}^n V_s + V_t \quad (5)$$

After obtaining individual tree volume (Volume.m3), it was then tabulated against the variables - height in meter (Height.m) and the diameter at breast height in centimeter (DBH.cm).

4. The Dataset used for modeling volume of *Quercus lanata*

A total of 49 trees have felled and collected data for developing merchantable volume equation for *Quercus lanata*. The data (diameter and height) has been collected as part of biomass equation development field work, carried out by Biomass Equation Development Team of RDC, as part of the project implemented by Forest Resources Management Division (FRMD). The data has been collected from all four regions – eastern, eastern central, western and western central. Summary of the data is presented below, while the detailed dataset used for this study is provided as an annexure.

4.1 Summary descriptive statistics of *Quercus lanata* dataset

> summary(ql)

Tree.ID	Height.m	DBH.cm	Volume.m3
qle01 : 1	Min. : 6.00	Min. :13.00	Min. :0.03199
qle02 : 1	1st Qu.:13.90	1st Qu.:28.30	1st Qu.:0.44575
qle03 : 1	Median :17.95	Median :46.10	Median :0.88245
qle04 : 1	Mean :18.19	Mean :46.31	Mean :1.44455
qle05 : 1	3rd Qu.:20.70	3rd Qu.:63.00	3rd Qu.:2.00375
qle06 : 1	Max. :28.90	Max. :91.50	Max. :5.88285

BA.m2	BAH.m3	DBH2H.m3
Min. :0.01327	Min. : 0.08336	Min. : 0.1061
1st Qu.:0.06290	1st Qu.: 1.06241	1st Qu.: 1.3527
Median :0.16691	Median : 2.60664	Median : 3.3189
Mean :0.20049	Mean : 4.25045	Mean : 5.4118
3rd Qu.:0.31172	3rd Qu.: 6.83544	3rd Qu.: 8.7032
Max. :0.65756	Max. :18.21427	Max. :23.1911

5. Fitting the models

The models have been fitted in R, which is a robust statistical computing environment. It is a powerful tool which provides wide range of statistical and graphical options to explore, calculate and manage data besides modelling. It is very powerful and widely used statistical tool which is free and allows user to customize the scripts depending on desired output, which is not possible in many of the statistical softwares.

After reading in the excel files into R, we created other variables namely; basal area in square meter (BA.m2), basal area in meter times height in meter (BAH.m3) and square of the diameter in meter times height in meter (DBH2H.m3). The height in meter (Height.m) and diameter in centimeter (DBH.cm) were measured and recorded in the field.

Prior to fitting models, we explored and examined each set of data by preparing descriptive summaries that provided mean, median and range of response and predictor variables. Then we plotted scatter graphs which provided sense of relationship between the response (volume) and predictor variables (namely DBH.cm, BA.m2, DBH2H.m3 and BAH.m3). These graphs showed curvilinear relationship between response and predictor variables. The scatter plots also clearly revealed the presence of phenomenon, referred in statistical parlance, as heteroscedasticity, which is the increase in variation in response (volume) variable with increase in value of the predictor variables.

Therefore, we fitted the models using the `gls ()` function of the `nlme` package of R, because the `gls ()` function has the capability to model heteroscedasticity. We didn't transform the variables, mainly response variable, because transformation makes it difficult to directly interpret the relationship between response and predictor variables; and secondly to compare the AIC and BIC values among the different models, the response variables need to be identical.

The models were fitted with volume as a function of four variables;

- 1) DBH.cm,
- 2) BA.m2,
- 3) DBH2H.m3 and
- 4) BAH.m3.

For each of the variable, we fitted one simple `gls ()` function, which can be written in the following form;

$$Y = \beta_0 + \beta_1 X + \epsilon, \quad (6)$$

Where $Y = \text{Volume (V)}$ and $X = \text{predictor variable}$

And then fitted 3 models with restricted natural cubic spline functions. The restricted natural cubic spline function enables better tracking of curvilinear relationship between response and predictor variables. These models introduce an additional predictor variable as part of a 3 knot-cubic spline. They take the following forms;

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon, \quad (7)$$

Where $Y = \text{Response variable, volume (V)}$

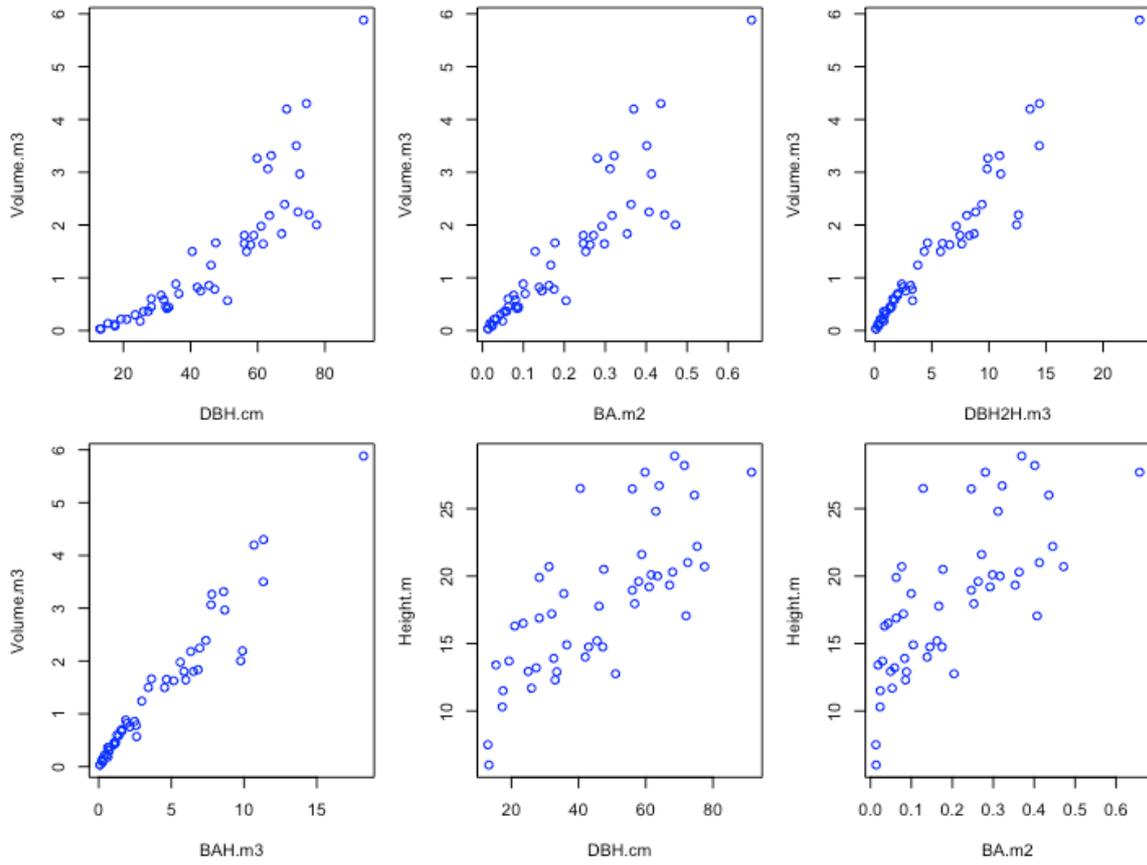
$X_1 = \text{Predictor variable}$

$X_2 = g(X_1)$

And $g(X_1)$ is the spline transformation of X_1 predictor variable

6. Summary Plots

Quercus lanata (N = 49)



7. Models and results

7.1 Model 1 - Volume with diameter at breast height (DBH) as predictor

```
> ql.m1 <- gls(Volume.m3 ~ DBH.cm)
> summary(ql.m1)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ DBH.cm

Data: NULL

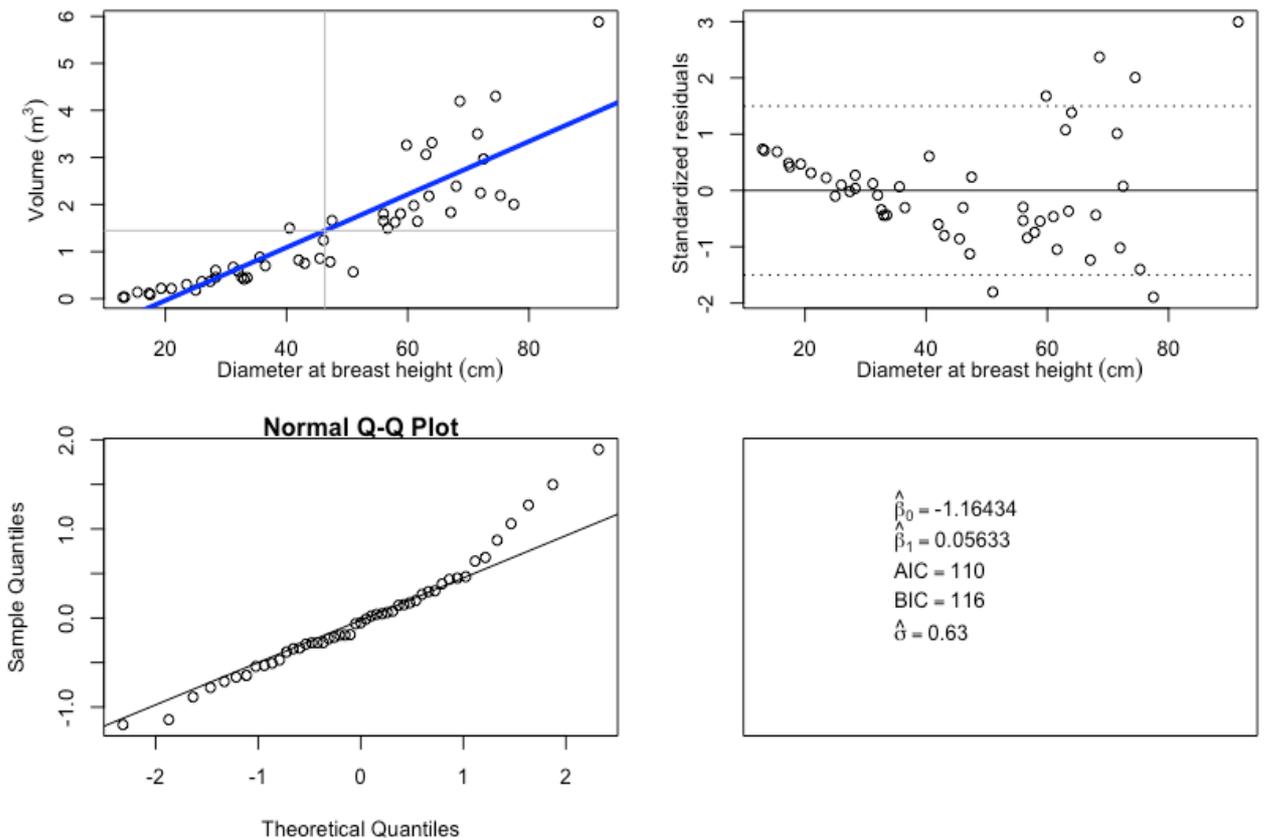
	AIC	BIC	logLik
	110.0369	115.5874	-52.01847

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-1.1643400	0.22585859	-5.155173	0
DBH.cm	0.0563327	0.00447031	12.601525	0

Plot of model 1

Q_lanata:Model 1: (Volume ~ dbh)



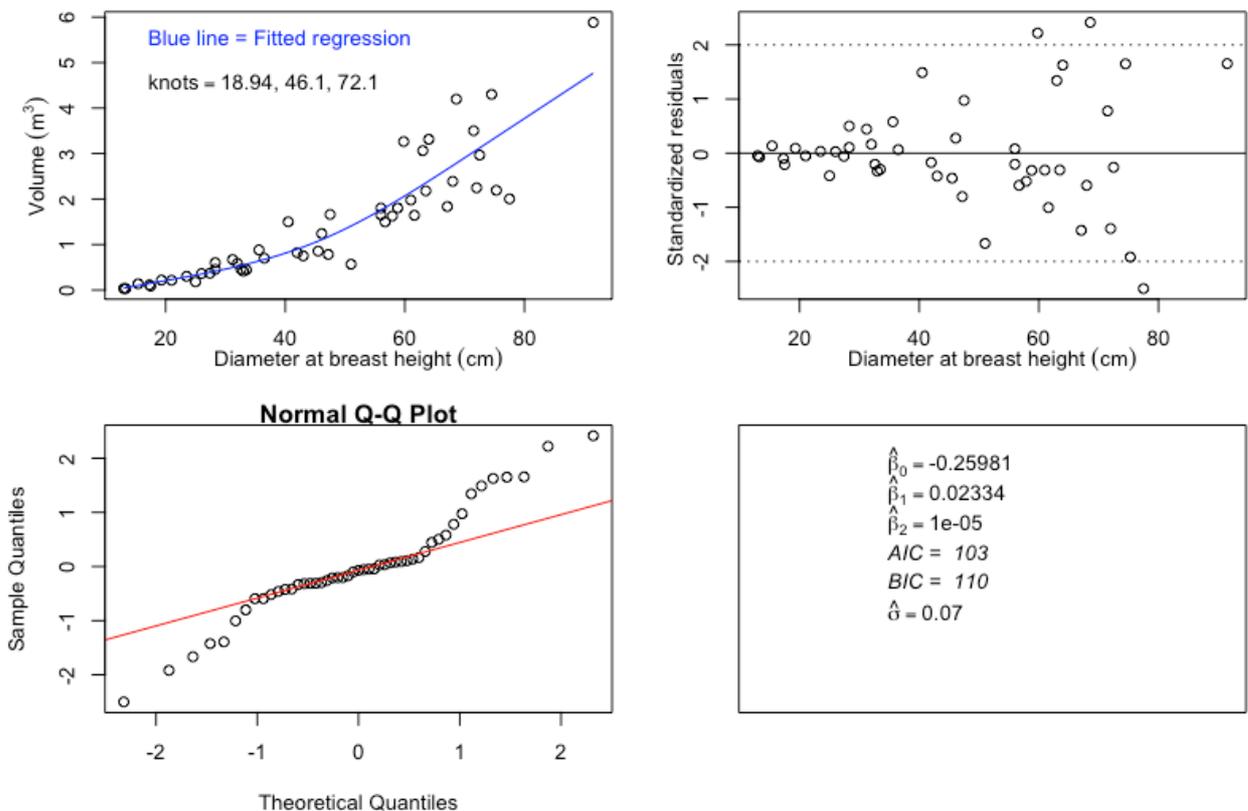
7.2 Model 2 - Volume with diameter at breast height (DBH) as predictor, with varFixed

```
> ql.m2 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints,
               na.action=na.omit, weights = varFixed(~DBH.cm))
> summary(ql.m2)
```

```
Generalized least squares fit by REML
Model: Volume.m3 ~ DBH.cm + DBH.cm.splinepoints
Data: NULL
      AIC      BIC    logLik
103.1108 110.4254 -47.55542
```

```
Variance function:
Structure: fixed weights
Formula: ~DBH.cm
```

```
Coefficients:
                Value Std.Error  t-value p-value
(Intercept)    -0.25981382 0.20210130 -1.285562  0.2050
DBH.cm          0.02334083 0.00746457  3.126882  0.0031
DBH.cm.splinepoints 0.00001457 0.00000367  3.974472  0.0002
```

Plot of Model 2**Q_lanata:Model 2 : (Volume ~ dbh), Cubic spline with varFixed**

7.3 Model 3- Volume with diameter at breast height (DBH) as predictor, with varPower

```
> ql.m3 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints,
  na.action=na.omit, weights = varPower(form =
  ~DBH.cm))
> summary(ql.m3)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ DBH.cm + DBH.cm.splinepoints

Data: NULL

	AIC	BIC	logLik
	61.46195	70.60516	-25.73098

Variance function:

Structure: Power of variance covariate

Formula: ~DBH.cm

Parameter estimates:

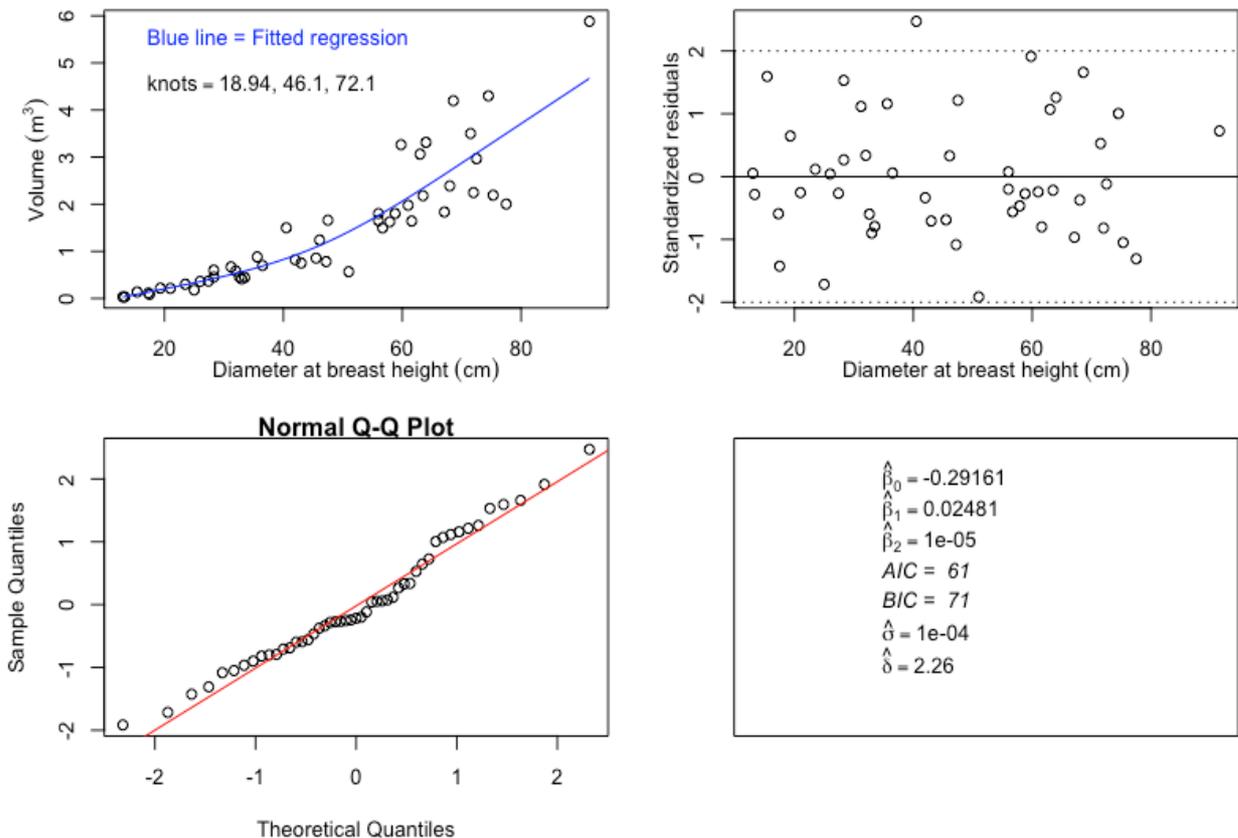
power
2.263931

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.29161222	0.03796676	-7.680724	0
DBH.cm	0.02481228	0.00225829	10.987195	0
DBH.cm.splinepoints	0.00001358	0.00000251	5.411975	0

Plot of Model 3

Q_lanata:Model 3: (Volume ~ dbh), Cubic spline with varPower



7.4 Model 4- Volume with diameter at breast height (DBH) as predictor, with varConstPower

```
> ql.m4 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints,
  na.action=na.omit, weights = varConstPower(form =
  ~DBH.cm))
> summary(ql.m4)
```

Generalized least squares fit by REML
 Model: Volume.m3 ~ DBH.cm + DBH.cm.splinepoints
 Data: NULL
 AIC BIC logLik
 63.4621 74.43395 -25.73105

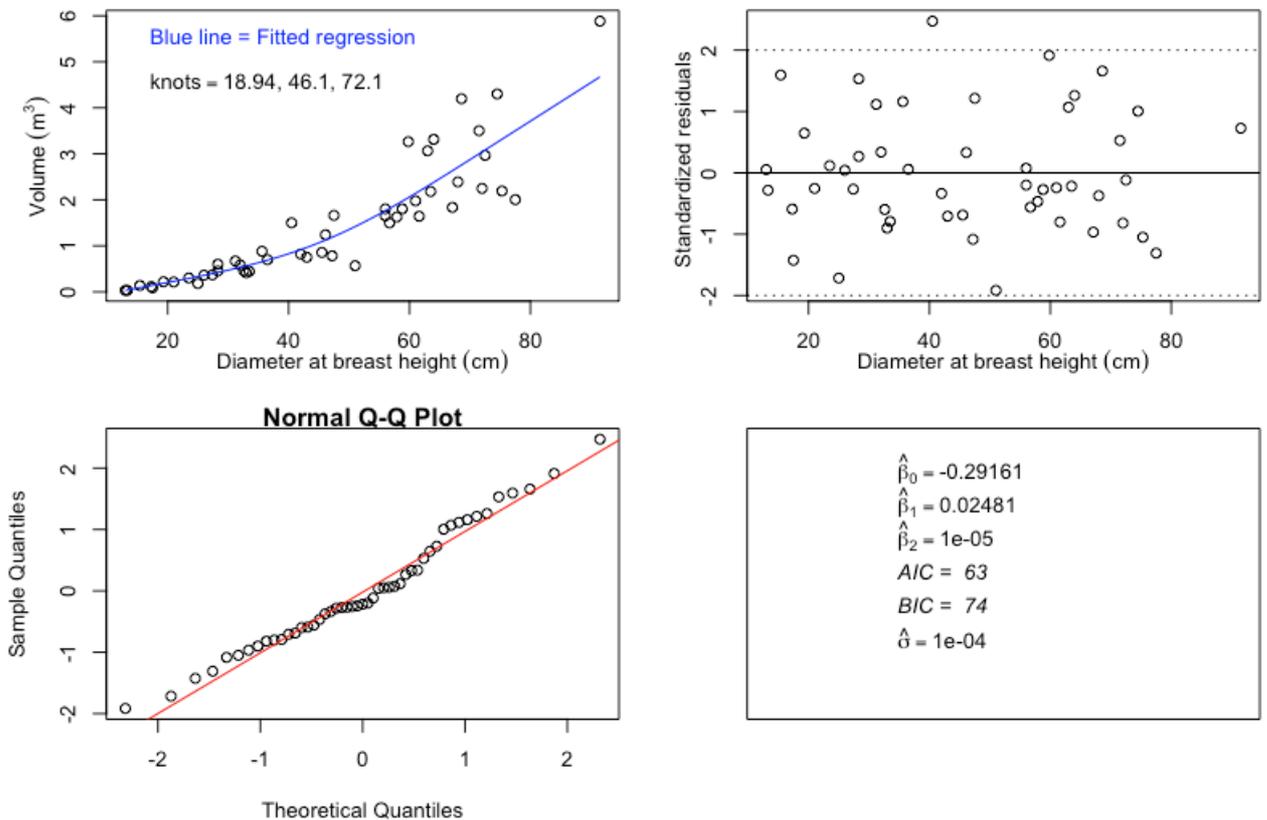
Variance function:
 Structure: Constant plus power of variance covariate
 Formula: ~DBH.cm
 Parameter estimates:
 const power
 0.09788284 2.26404430

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.29161283	0.03796817	-7.680455	0
DBH.cm	0.02481230	0.00225830	10.987138	0
DBH.cm.splinepoints	0.00001358	0.00000251	5.411880	0

Plot of Model 4

Q_lanata:Model 4: (Volume ~ dbh), Cubic spline with varConstPower



7.5 Model 5 - Volume with basal area (BA) as predictor

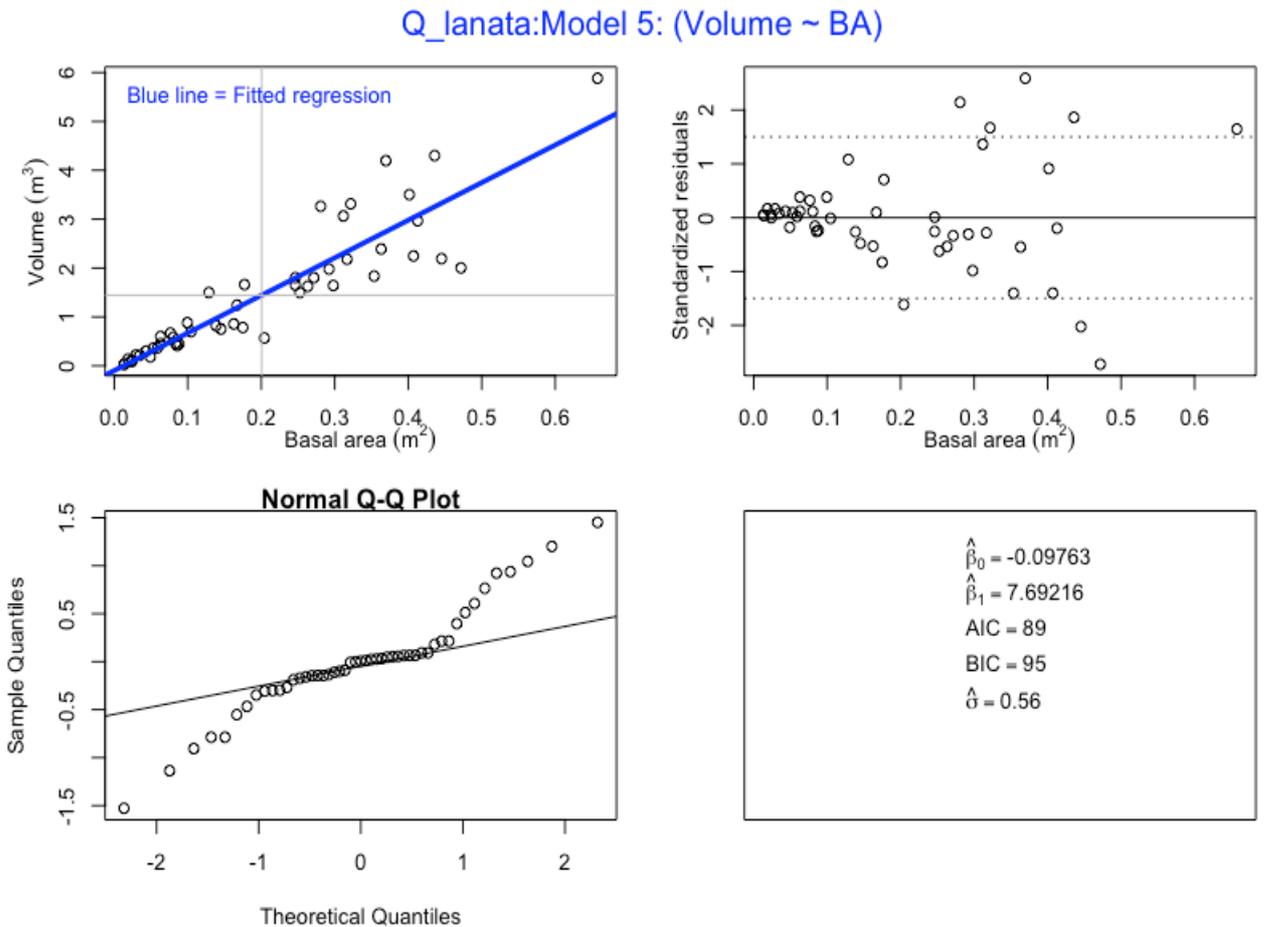
```
> ql.m5 <- gls(Volume.m3 ~ BA.m2)
> summary(ql.m5)
```

Generalized least squares fit by REML

```
Model: Volume.m3 ~ BA.m2
Data: NULL
      AIC      BIC    logLik
89.01812 94.56856 -41.50906
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.097634	0.1323154	-0.737892	0.4642
BA.m2	7.692158	0.5252917	14.643594	0.0000

Plot of Model 5

7.6 Model 6 - Volume with basal area (BA) as predictor, with varFixed

```
> ql.m6<- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
              na.action=na.omit, weights = varFixed(~BA.m2))
> summary(ql.m6)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ BA.m2 + BA.m2.splinepoints

Data: NULL

	AIC	BIC	logLik
	43.52879	50.84336	-17.7644

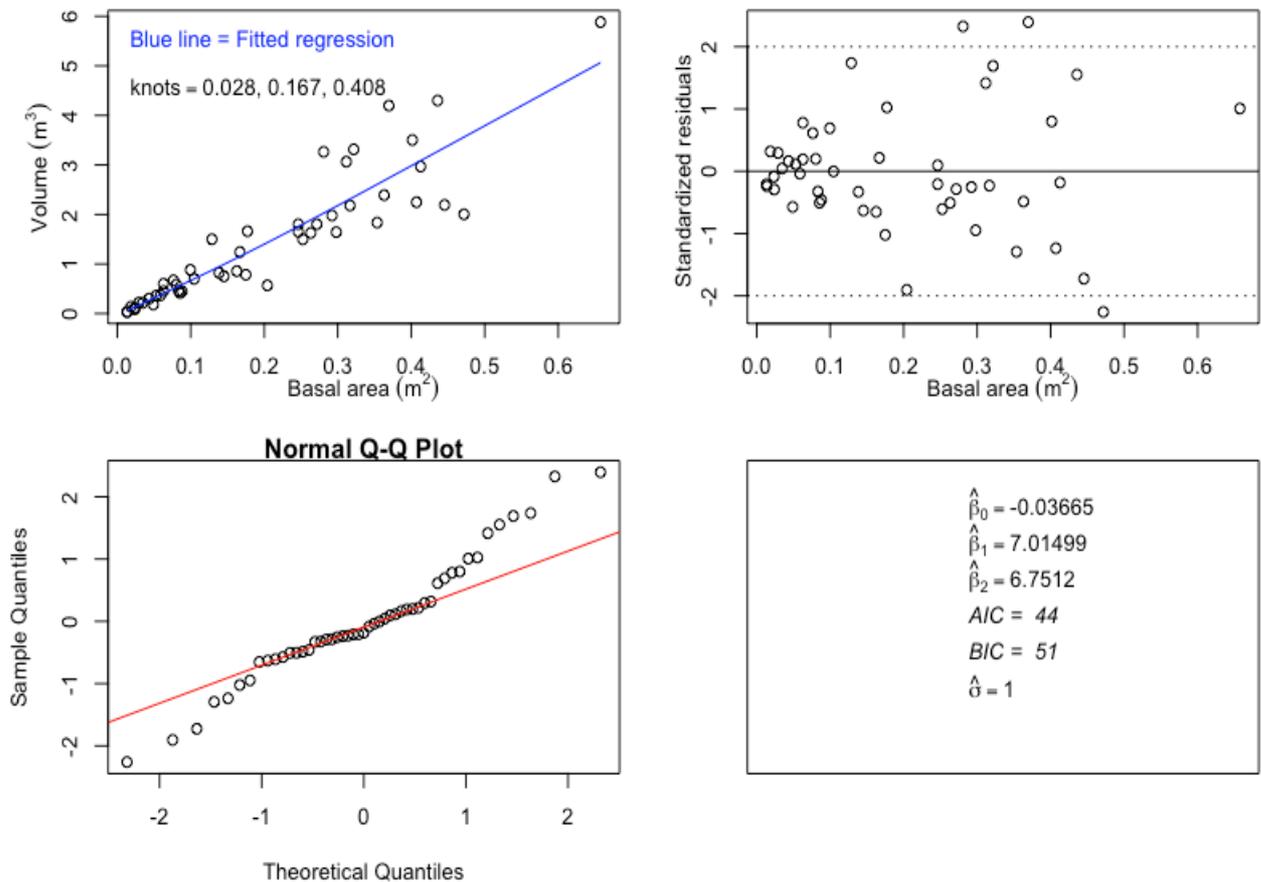
Variance function:

Structure: fixed weights

Formula: ~BA.m2

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.036645	0.061911	-0.591904	0.5568
BA.m2	7.014993	0.968807	7.240858	0.0000
BA.m2.splinepoints	6.751196	12.594779	0.536031	0.5945

Plot of Model 6**Q_lanata:Model 6: (Volume ~ BA), Cubic spline with varFixed**

7.7 Model 7 Volume with basal area (BA) as predictor, with varPower

```
> ql.m7 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
               na.action=na.omit, weights = varPower(form = ~BA.m2))
> summary(ql.m7)
```

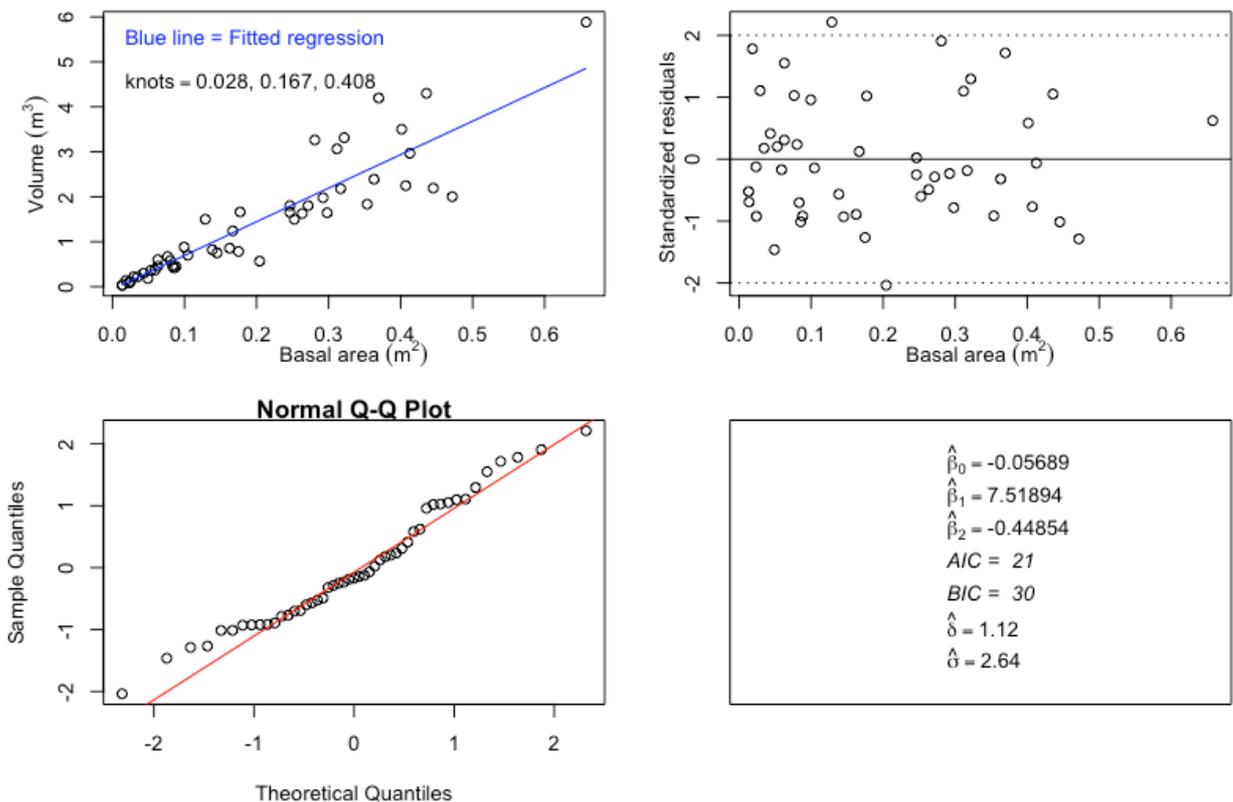
```
Generalized least squares fit by REML
Model: Volume.m3 ~ BA.m2 + BA.m2.splinepoints
Data: NULL
      AIC      BIC    logLik
20.94348 30.08669 -5.471739
```

```
Variance function:
Structure: Power of variance covariate
Formula: ~BA.m2
Parameter estimates:
  power
1.120354
```

```
Coefficients:
                Value Std.Error  t-value p-value
(Intercept)    -0.056893  0.017482 -3.254320  0.0021
BA.m2           7.518939  0.588396 12.778697  0.0000
BA.m2.splinepoints -0.448535 11.420781 -0.039274  0.9688
```

Plot of Model 7

Q_lanata:Model 7: (Volume ~ BA), Cubic spline with varPower



7.8 Model 8 – Volume with basal area (BA) as predictor, with varConstPower

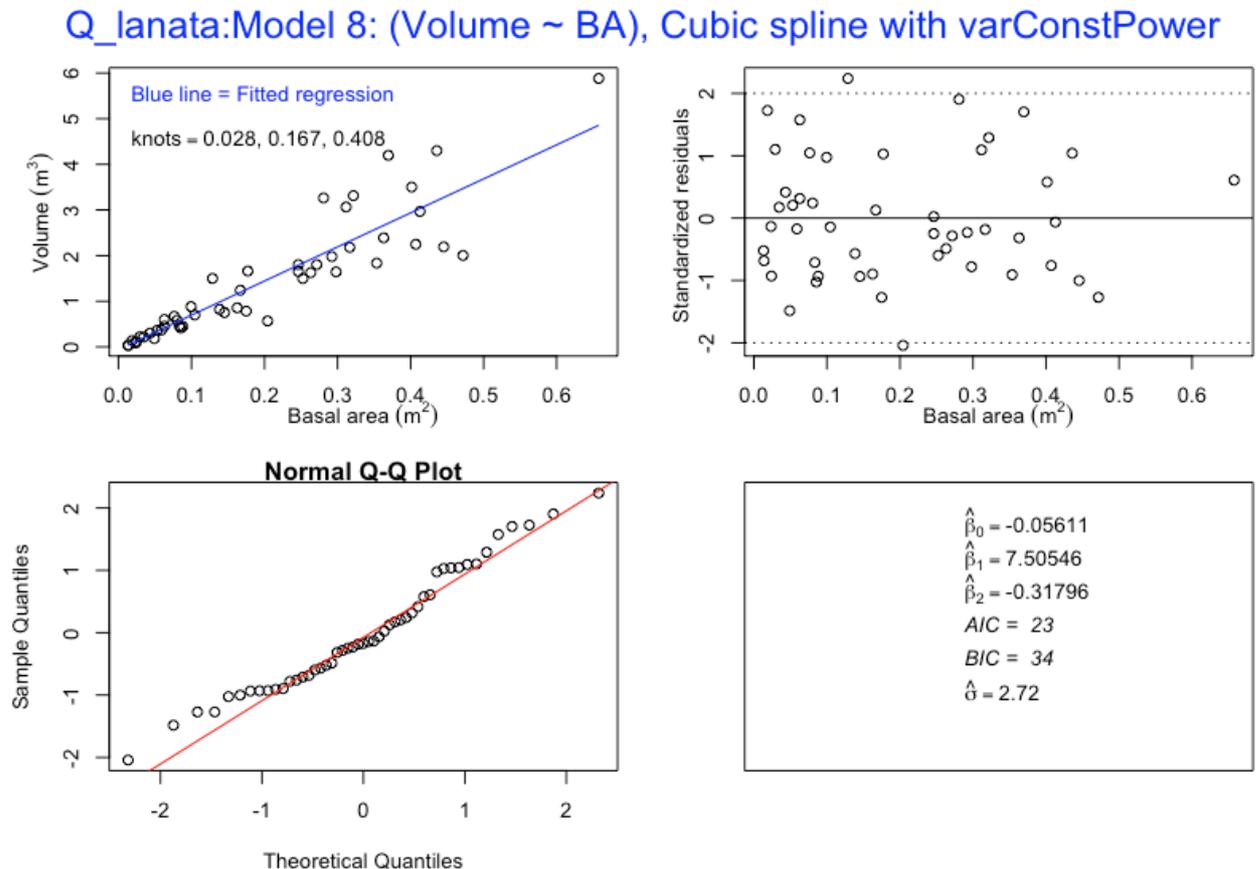
```
> ql.m8 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
               na.action=na.omit, weights = varConstPower(form =
               ~BA.m2))
> summary(ql.m8)
```

```
Generalized least squares fit by REML
Model: Volume.m3 ~ BA.m2 + BA.m2.splinepoints
Data: NULL
      AIC      BIC    logLik
22.9291 33.90095 -5.464551
```

```
Variance function:
Structure: Constant plus power of variance covariate
Formula: ~BA.m2
Parameter estimates:
      const      power
0.001011173 1.145436177
Coefficients:
```

	Value	Std.Error	t-value	p-value
(Intercept)	-0.056107	0.018002	-3.116790	0.0031
BA.m2	7.505464	0.587958	12.765298	0.0000
BA.m2.splinepoints	-0.317960	11.480647	-0.027695	0.9780

Plot of Model 8



7.9 Model 9 – Volume with square of diameter at breast height * height (DBH2H) as predictor

```
> ql.m9 <- gls(Volume.m3 ~ DBH2H.m3)
> summary(ql.m9)
```

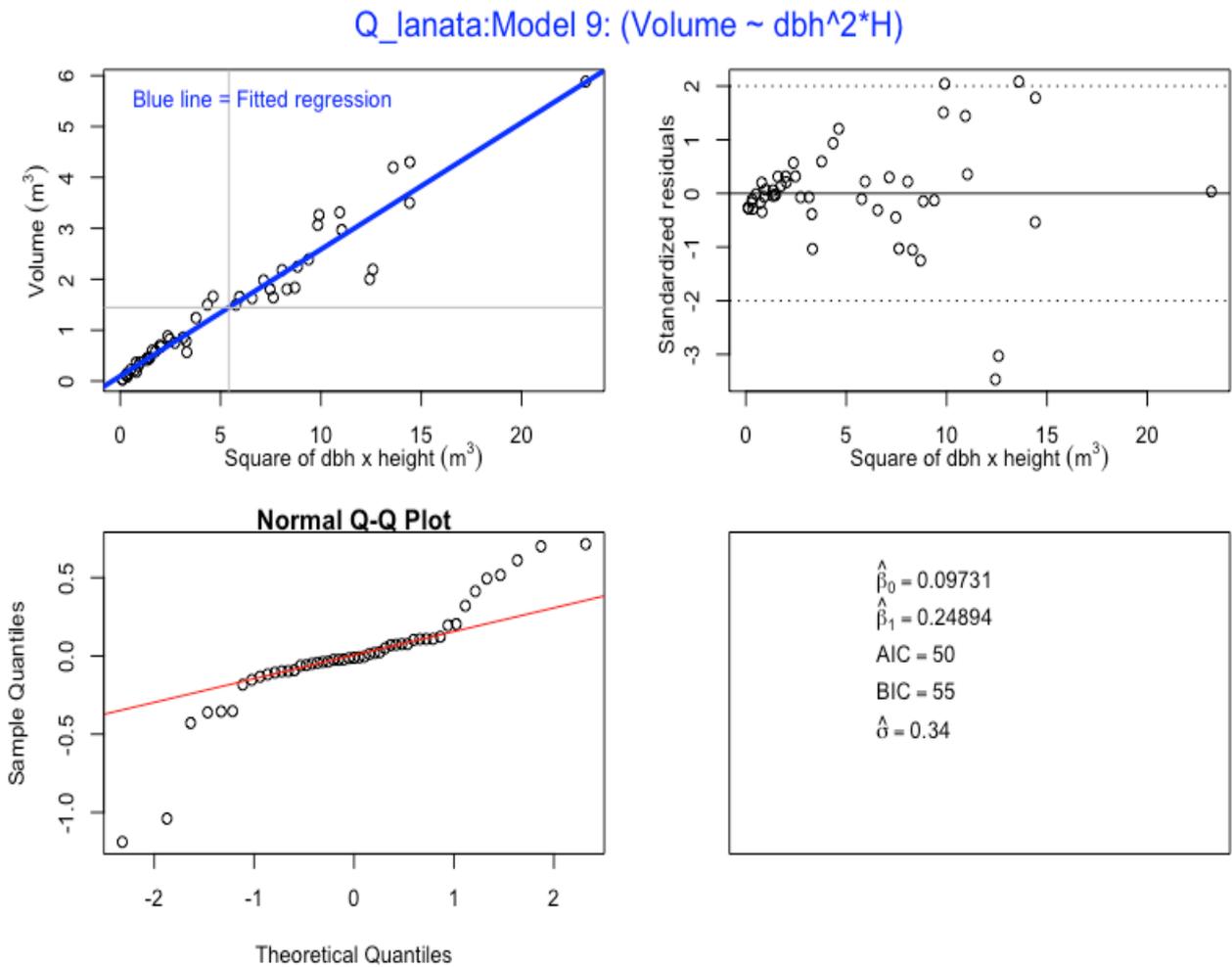
Generalized least squares fit by REML

Model: Volume.m3 ~ DBH2H.m3
 Data: NULL
 AIC BIC logLik
 49.77417 55.32461 -21.88708

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.09730642	0.07198822	1.351699	0.1829
DBH2H.m3	0.24894391	0.00974816	25.537528	0.0000

Plot of Model 9



7.10 Model 10 – Volume with square of diameter at breast height * height (DBH2H) as predictor, with varFixed

```
> ql.m10 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,
                na.action=na.omit, weights = varFixed(~DBH2H.m3))
> summary(ql.m10)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints

Data: NULL

	AIC	BIC	logLik
	15.41372	22.72829	-3.706862

Variance function:

Structure: fixed weights

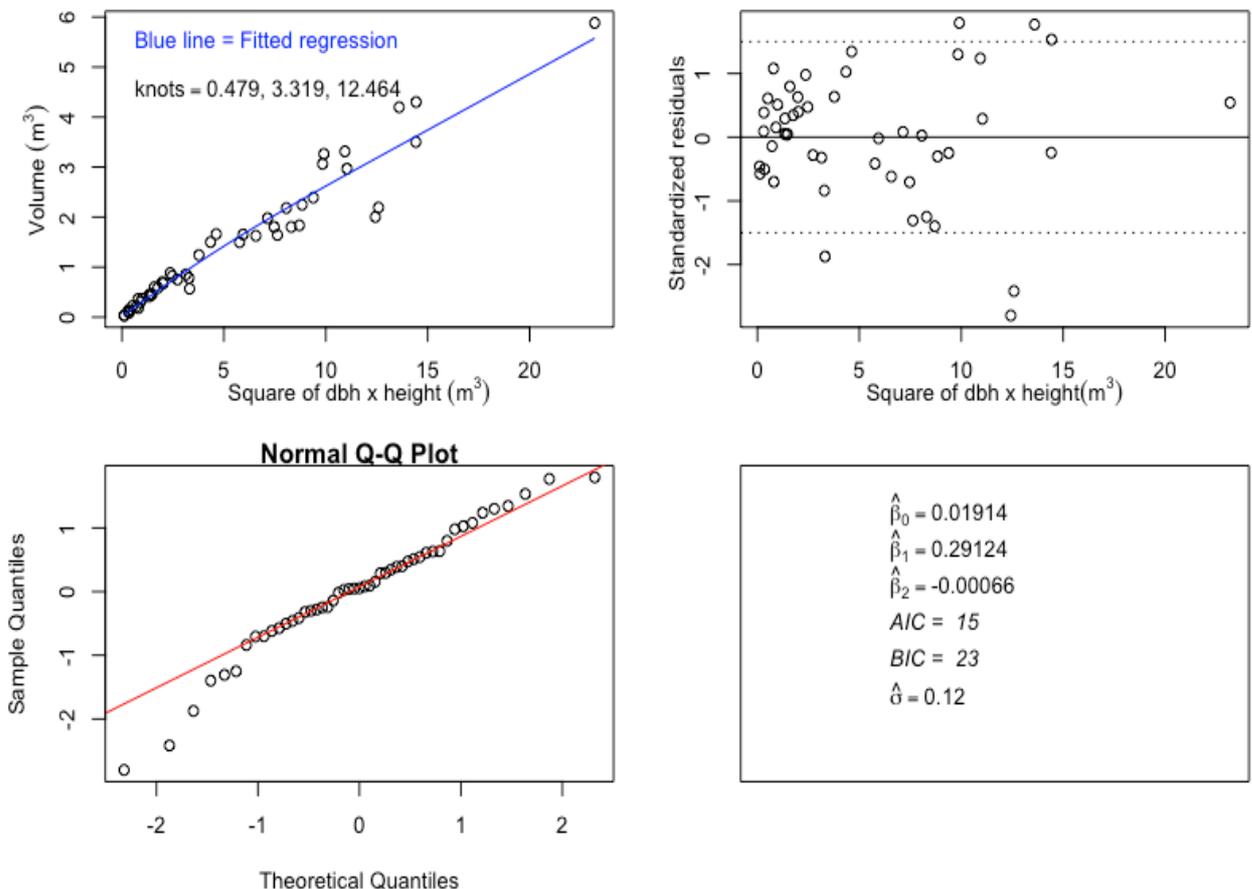
Formula: ~DBH2H.m3

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.01913523	0.022484547	0.851039	0.3992
DBH2H.m3	0.29124163	0.019295525	15.093739	0.0000
DBH2H.m3.splinepoints	-0.00066041	0.000368112	-1.794043	0.0794

Plot of Model 10

Q_lanata:Model 10: (Volume ~ dbh²*H), Cubic Spline with varFixed



7.11 Model 11– Volume with square of diameter at breast height * height (DBH2H) as predictor, with varPower

```
> ql.m11 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,
  na.action=na.omit, weights = varPower(form =
  ~DBH2H.m3))
> summary(ql.m11)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints

Data: NULL

AIC	BIC	logLik
1.704271	10.84748	4.147865

Variance function:

Structure: Power of variance covariate

Formula: ~DBH2H.m3

Parameter estimates:

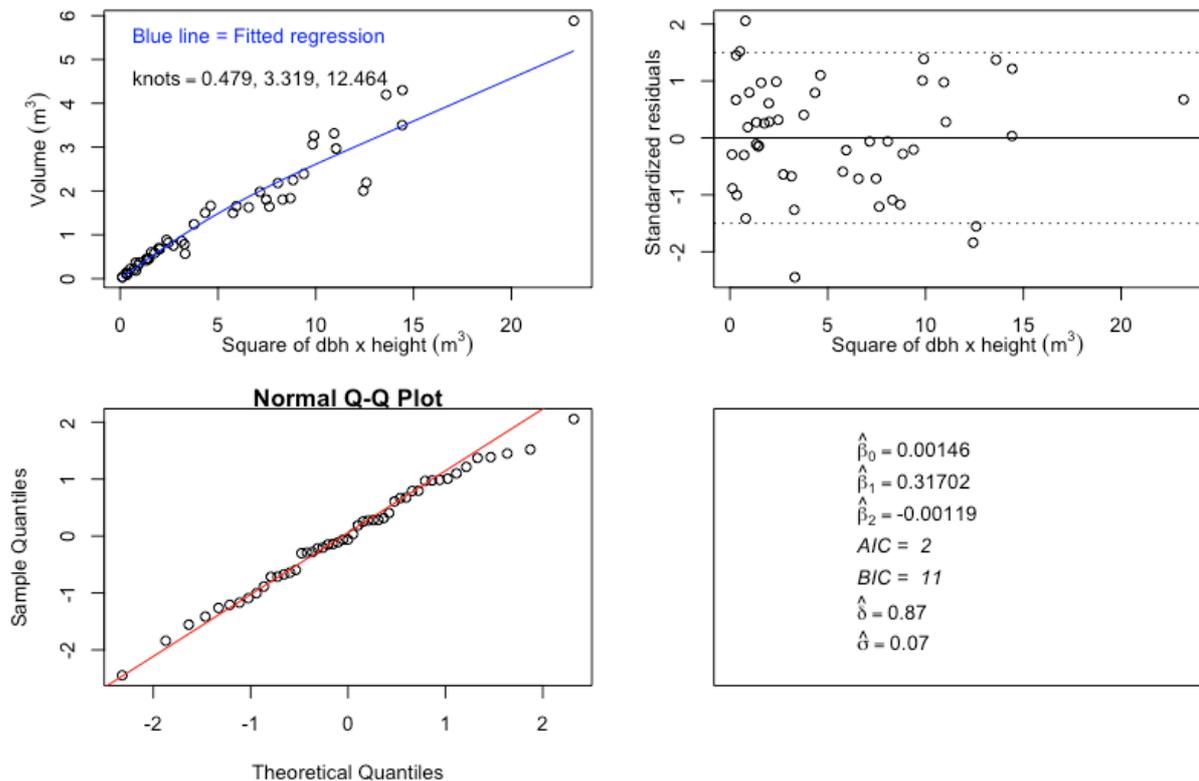
power
0.8709407

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0014647	0.007142576	0.205063	0.8384
DBH2H.m3	0.3170159	0.014450285	21.938386	0.0000
DBH2H.m3.splinepoints	-0.0011887	0.000360381	-3.298497	0.0019

Plot of Model 11

Q_lanata:Model 11: (Volume ~ dbh²*H), Cubic Spline with varPower



7.12 Model 12 –Volume with square of diameter at breast height * height (DBH2H) as predictor, with varConstPower

```
> ql.m12 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,
  na.action=na.omit, weights = varConstPower(form =
  ~DBH2H.m3))
> summary(ql.m12)
```

Generalized least squares fit by REML

```
Model: Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints
Data: NULL
      AIC      BIC    logLik
3.399104 14.37095 4.300448
```

Variance function:

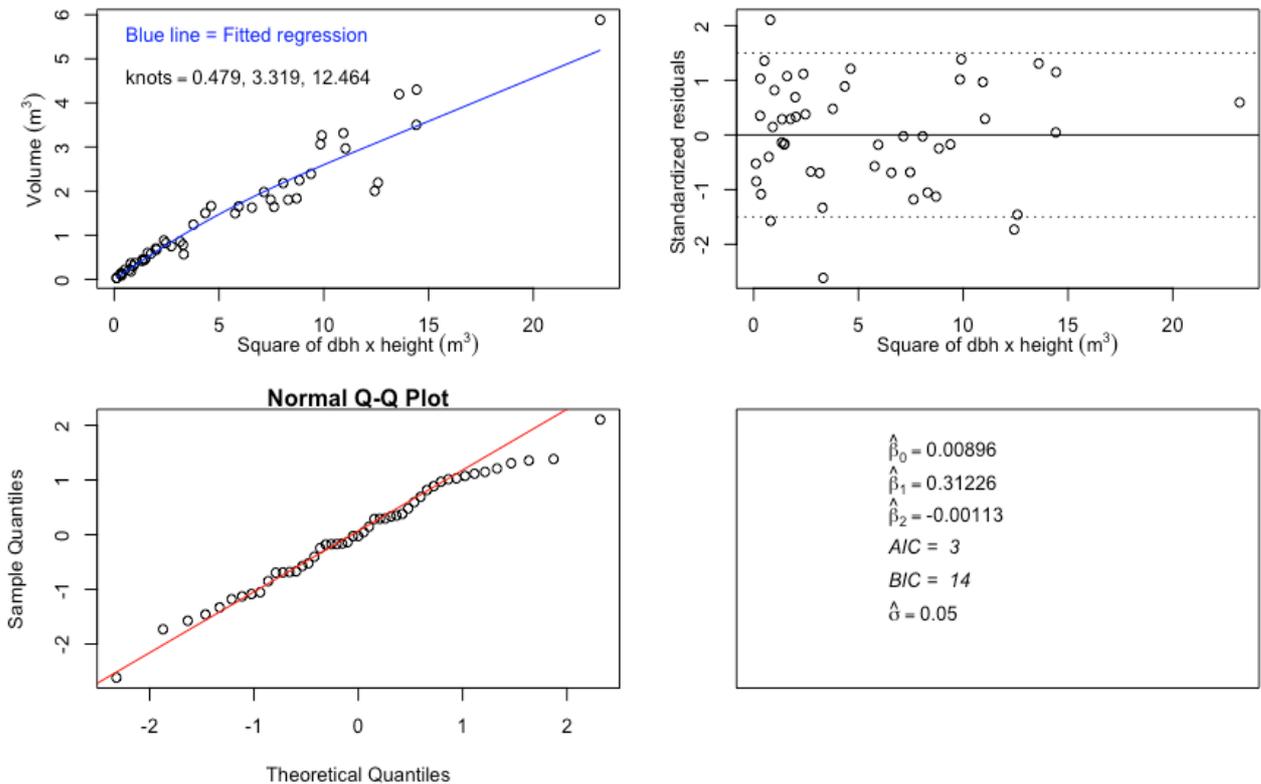
```
Structure: Constant plus power of variance covariate
Formula: ~DBH2H.m3
Parameter estimates:
      const      power
0.2928089 1.0166474
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.00895585	0.011286346	0.793512	0.4316
DBH2H.m3	0.31225586	0.014902564	20.953164	0.0000
DBH2H.m3.splinepoints	-0.00113361	0.000376344	-3.012173	0.0042

Plot of Model 12

Q_lanata:Model 12: (Volume ~ dbh²*H), Cubic Spline with varConstPower



7.13 Model 13 – Volume with basal area * height (BAH) as predictor

```
> ql.m13 <- gls(Volume.m3 ~ BAH.m3)
> summary(ql.m13)
```

Generalized least squares fit by REML

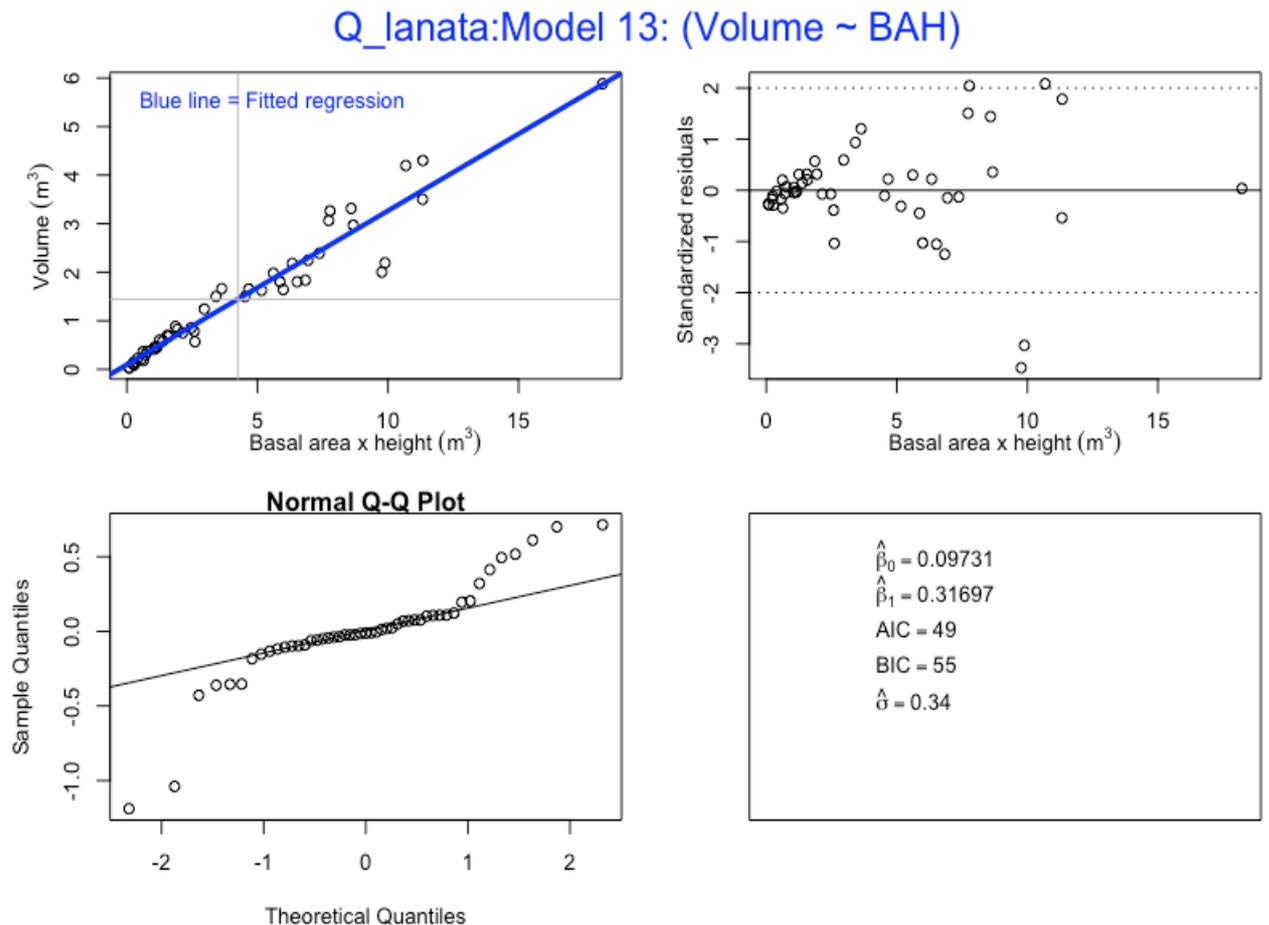
Model: Volume.m3 ~ BAH.m3

Data: NULL

	AIC	BIC	logLik
	49.29104	54.84148	-21.64552

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0973064	0.07198822	1.351699	0.1829
BAH.m3	0.3169652	0.01241174	25.537528	0.0000

Plot of Model 13

7.14 Model 14 – Volume with basal area * height (BAH) as predictor, with varFixed

```
> ql.m14 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
                na.action=na.omit, weights = varFixed(~BAH.m3))
> summary(ql.m14)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints

Data: NULL

	AIC	BIC	logLik
	13.48121	20.79577	-2.740604

Variance function:

Structure: fixed weights

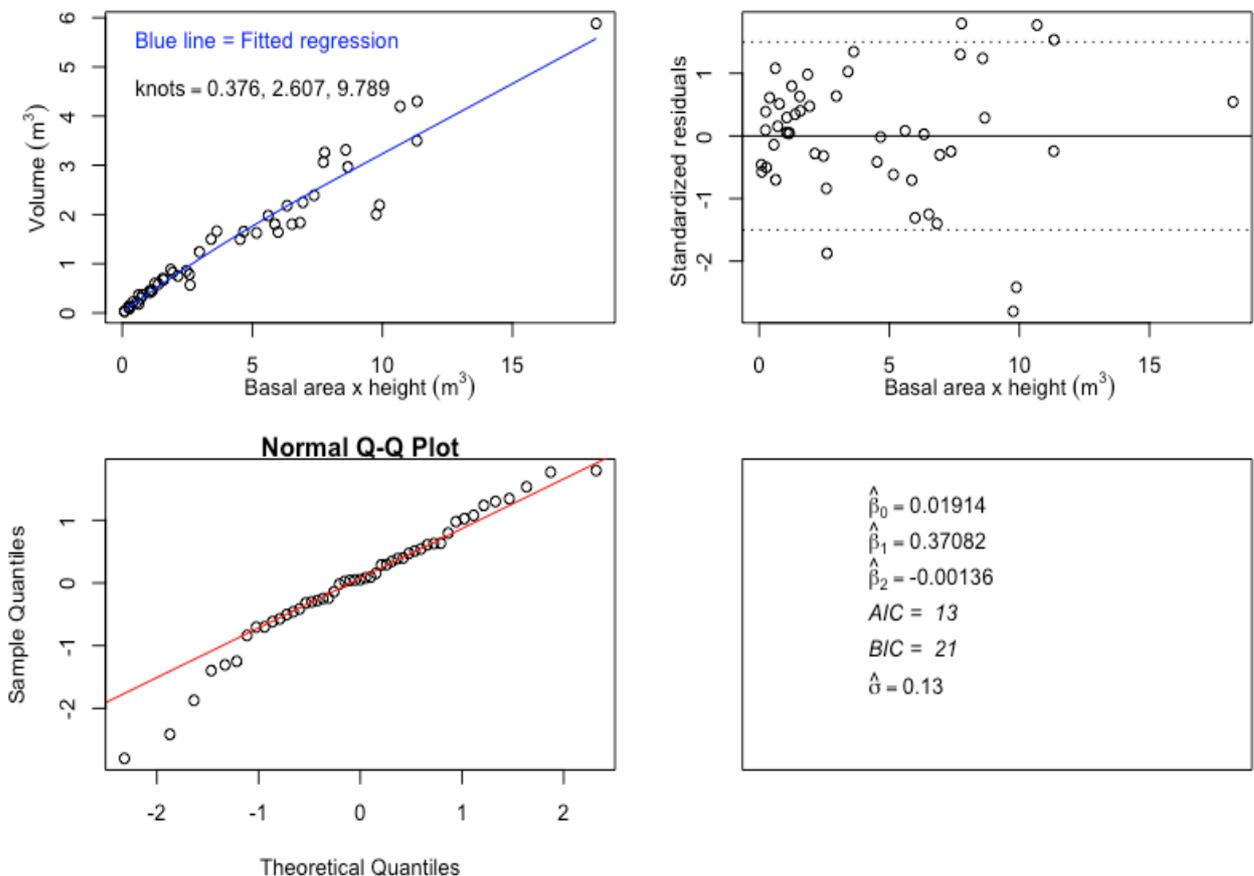
Formula: ~BAH.m3

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0191352	0.022484547	0.851039	0.3992
BAH.m3	0.3708204	0.024567826	15.093739	0.0000
BAH.m3.splinepoints	-0.0013631	0.000759819	-1.794043	0.0794

Plot of Model 14

Q_lanata:Model 14: (Volume ~ BAH), Cubic spline with varFixed



7.15 Model 15– Volume with basal area * height (BAH) as predictor, with varPower

```
> ql.m15 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
  na.action=na.omit, weights = varPower(form =
  ~BAH.m3))
> summary(ql.m15)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints

Data: NULL

	AIC	BIC	logLik
	-0.2282451	8.914962	5.114123

Variance function:

Structure: Power of variance covariate

Formula: ~BAH.m3

Parameter estimates:

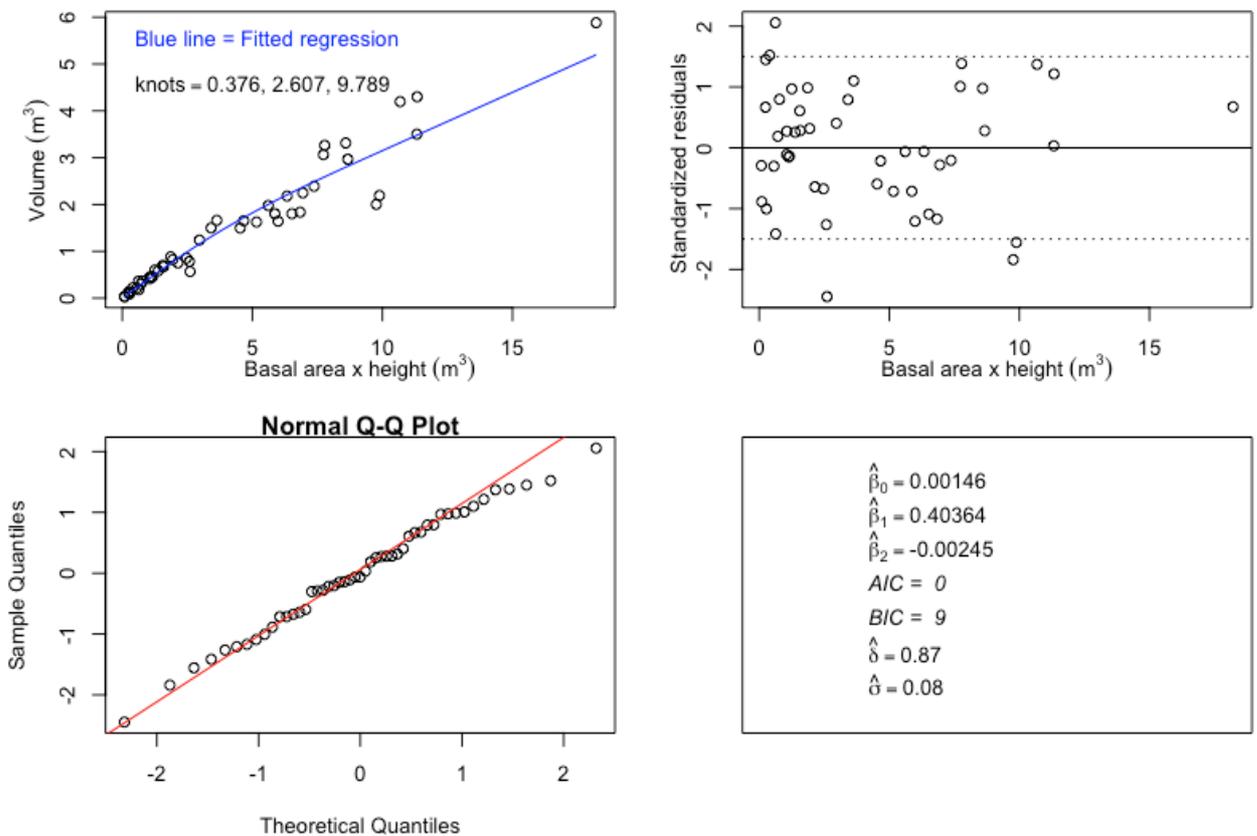
power
0.8709407

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0014647	0.007142576	0.205063	0.8384
BAH.m3	0.4036372	0.018398674	21.938387	0.0000
BAH.m3.splinepoints	-0.0024536	0.000743862	-3.298497	0.0019

Plot of Model 15

Q_lanata:Model 15: (Volume ~ BAH), Cubic spline with varPower



7.16 Model 16 – Volume with basal area * height (BAH) as predictor, with varConstPower

```
> ql.m16 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
  na.action=na.omit, weights = varConstPower(form =
  ~BAH.m3))
> summary(ql.m16)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints

Data: NULL

	AIC	BIC	logLik
	1.466588	12.43844	5.266706

Variance function:

Structure: Constant plus power of variance covariate

Formula: ~BAH.m3

Parameter estimates:

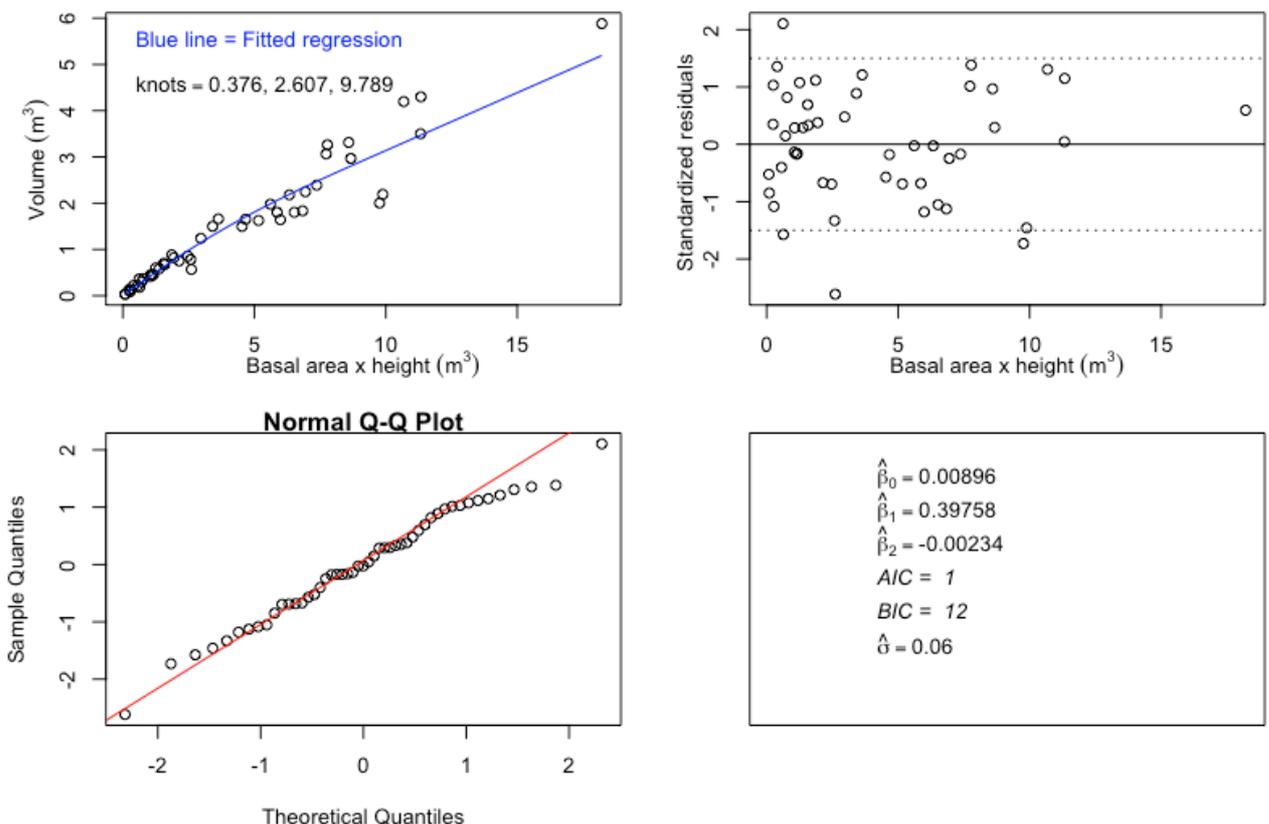
	const	power
	0.2290484	1.0166475

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0089558	0.011286340	0.793511	0.4316
BAH.m3	0.3975765	0.018974532	20.953167	0.0000
BAH.m3.splinepoints	-0.0023399	0.000776812	-3.012174	0.0042

Plot of Model 16

Q_lanata:Model 16: (Volume ~ BAH), Cubic spline with varConstPower



8. Model evaluation using AIC and BIC values

SN	Model	AIC	BIC
1	Model 1 > ql.m1 <- gls(Volume.m3 ~ DBH.cm)	110	116
2	Model 2 > ql.m2 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints, na.action=na.omit, weights = varFixed(~DBH.cm))	103	110
3	Model 3 > ql.m3 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints, na.action=na.omit, weights = varPower(form = ~DBH.cm))	61	71
4	Model 4 > ql.m4 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints, na.action=na.omit, weights = varConstPower(form = ~DBH.cm))	63	74
5	Model 5 > ql.m5 <- gls(Volume.m3 ~ BA.m2)	89	95
6	Model 6 > ql.m6<- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights = varFixed(~BA.m2))	44	51
7	Model 7 > ql.m7 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights = varPower(form = ~BA.m2))	21	30
8	Model 8 > ql.m8 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights = varConstPower(form = ~BA.m2))	23	34
9	Model 9 > ql.m9 <- gls(Volume.m3 ~ DBH2H.m3)	50	55
10	Model 10 > ql.m10 <-gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints, na.action=na.omit, weights = varFixed(~DBH2H.m3))	15	23

11	Model 11 > ql.m11 <-gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints, na.action=na.omit, weights = varPower(form = ~DBH2H.m3))	2	11
12	Model 12 > ql.m12 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints, na.action=na.omit, weights = varConstPower(form = ~DBH2H.m3))	3	14
13	Model 13 > ql.m13 <- gls(Volume.m3 ~ BAH.m3)	49	55
14	Model 14 > ql.m14 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints, na.action=na.omit, weights = varFixed(~BAH.m3))	13	21
15	Model 15 > ql.m15 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints, na.action=na.omit, weights = varPower(form = ~BAH.m3))	0	9
16	Model 16 > ql.m16 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints, na.action=na.omit, weights = varConstPower(form = ~BAH.m3))	1	12

9. Selected Models

The best fitting models have been selected based on Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) values of the fitted models. The BIC value was mainly relied upon as it imposes a stronger penalty for the number of parameters in the model that need to be estimated. Smaller the values of AIC and BIC, better the fit of the model. Therefore, for *Quercus lanata*, the selected models are;

1. Model 7 (Model which doesn't use height)


```
ql.m7 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
              na.action=na.omit, weights = varPower(form = ~BA.m2))
```
2. Model 15 (Model which uses the height)


```
ql.m15 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
               na.action=na.omit, weights = varPower(form = ~BAH.m3))
```

Two models have been selected for *Quercus lanata*, one without height ($X_1 = BA$ which is model 7) and one with the height ($X_1 = BAH$, which is Model 15) as predictor or explanatory variable. Both the models have been fitted with natural (restricted) cubic spline function within a linear model framework. Although, nonlinear models are more flexible, they are more complicated than the linear models. The complications involved and amount of time and efforts spent on fitting nonlinear models often fail to justify by the improvements in the models. Moreover, the models fitted with natural (restricted) cubic spline functions perform well and track the curvilinearity better than nonlinear functions that were examined.

10. Demonstration of use of the selected best fit model

In general, the natural spline predictor with knots represented by t_1 , t_2 and t_3 takes the following form;

$$Y = \beta_0 + \beta_1 X + \beta_2 X_s + \varepsilon \quad (8)$$

Where X_s corresponds to value in X as follows:

$$X_s = g(X) = (X - t_1)_+^3 - (X - t_2)_+^3 \frac{(t_3 - t_1)}{(t_3 - t_2)} + (X - t_3)_+^3 \frac{(t_2 - t_1)}{(t_3 - t_2)} \quad (9)$$

and the value of the positive part functions depend on the values of the knots as follows;

$$(X - t_1)_+^3 = (X - t_1)_+^3, \text{ if } X > t_1 \text{ and } (X - t_1)_+^3 = 0, \text{ if } X < t_1 \quad (10)$$

$$(X - t_2)_+^3 = (X - t_2)_+^3, \text{ if } X > t_2, \text{ and } (X - t_2)_+^3 = 0, \text{ if } X < t_2 \quad (11)$$

$$(X - t_3)_+^3 = (X - t_3)_+^3, \text{ if } X > t_3, \text{ and } (X - t_3)_+^3 = 0, \text{ if } X < t_3 \quad (12)$$

Where t_1 , t_2 and t_3 for the above models are 10th, 50th and 90th percentiles and are called knots. The values of knots differ from species and models.

To demonstrate use of the selected models for *Quercus lanata* – model 7, the knots t_1 , t_2 and t_3 are 0.028, 0.167 and 0.408 as generated by the model. The model 7 has been fitted with volume as function of basal area in meter square (BA) i.e

$$BA = \pi r^2 \quad (13)$$

where in

$$r^2 = \left[\frac{dbh}{2 \times 100} \right]^2 \quad (14)$$

Where r is radius in meters and dbh is diameter at breast height in centimeters.

Therefore, *Quercus lanata* with diameter of 64 cm resulting in basal area of 0.321699087 m², the volume can be estimated using the above equation (model 7) as below. But first the value of BA.m₂ has to be calculated, which is;

$$\begin{aligned} BA &= \pi r^2 = \frac{\pi \cdot 64^2}{200^2} = 0.321699087 \text{ m}^2 \\ g(X) &= (X - t_1)_+^3 - (X - t_2)_+^3 \frac{(t_3 - t_1)}{(t_3 - t_2)} + (X - t_3)_+^3 \frac{(t_2 - t_1)}{(t_3 - t_2)} \\ g(BA) &= (BA - t_1)_+^3 - (BA - t_2)_+^3 \frac{(t_3 - t_1)}{(t_3 - t_2)} + (BA - t_3)_+^3 \frac{(t_2 - t_1)}{(t_3 - t_2)} \\ g(BA) &= (0.321699087 - 0.028)_+^3 - (0.321699087 - 0.167)_+^3 \frac{(0.408 - 0.028)}{(0.408 - 0.167)} + 0 \\ &= (0.293699087)_+^3 - (0.154699087)_+^3 \frac{(0.38)}{(0.241)} + 0 \\ &= (0.293699087)_+^3 - (0.154699087)_+^3 * 1.5767635 + 0 \\ &= 0.025334235 - 0.003702229 * 1.5767635 \\ &= 0.025334235 - 0.005837539 \\ &= 0.01949669 \end{aligned}$$

Hence, the volume predicted for this tree by the selected model (model 7) is

$$\begin{aligned} V &= \beta_0 + \beta_1 \cdot BA + \beta_2 BA \cdot m_2 + \varepsilon \\ &= -0.056893 + 7.518939 * 0.321699087 + (-0.448535 * 0.01949669) \\ &= -0.056893 + 2.4188358 - 0.00874495 \\ &= \mathbf{2.3531978 \text{ m}^3} \end{aligned}$$

Similarly, to demonstrate model 15 with t_1 , t_2 and t_3 of 0.376, 2.607 and 9.789 respectively, we considered this same tree but with height, i.e $dbh = 64$ cm resulting in $BA = 0.3216990877$ m² and height (H) = 26.7 m.

$$\begin{aligned} BAH &= 0.3216990877 \times 26.7 \\ &= 8.5893656 \\ g(X) &= (X - t_1)_+^3 - (X - t_2)_+^3 \frac{(t_3 - t_1)}{(t_3 - t_2)} + (X - t_3)_+^3 \frac{(t_2 - t_1)}{(t_3 - t_2)} \\ g(BAH) &= (BAH - t_1)_+^3 - (BAH - t_2)_+^3 \frac{(t_3 - t_1)}{(t_3 - t_2)} + (BAH - t_3)_+^3 \frac{(t_2 - t_1)}{(t_3 - t_2)} \end{aligned}$$

$$\begin{aligned}
&= (8.5893656 - 0.376)_+^3 - (8.5893656 - 2.607)_+^3 \frac{(9.789-0.376)}{(9.789-2.607)} + 0 \\
&= (8.2133656)_+^3 - (5.9823656)_+^3 \frac{(9.413)}{(7.182)} + 0 \\
&= (8.2133656)_+^3 - (5.9823656)_+^3 * 1.3106377 + 0 \\
&= 554.068514 - 214.101081 * 1.3106377 + 0 \\
&= 554.068514 - 304.6301631 \\
&= 273.4595645
\end{aligned}$$

Hence, the volume predicted by model 15 for this tree is;

$$\begin{aligned}
V &= \beta_0 + \beta_1 \cdot BAH \cdot m^3 + \beta_2 BAH \cdot m^3_2 + \varepsilon \\
&= 0.0014647 + 0.4036372 * 8.5893656 + (-0.0024536 * 273.4595645) \\
&= 0.0014647 + 3.4669875 + (-0.67096039) \\
&= \mathbf{2.79749181 \text{ m}^3}
\end{aligned}$$

The field measured volume for this particular tree with DBH of 64 cm and height of 26.7 m is 3.313997661 m³.

11. Model Performance

To assess the performance of selected models, we compared the volume predicted by selected models (7 and 15) with the volume of the tree as measured in the field. Using the equations of the selected models, volume prediction or estimation was done in R.

SN	Tree_ID	Height (in m)	DBH (in cm)	Volume in m ³ (Field measured) [A]	Predicted Volume Model_7 [B]	Predicted Volume Model_15 [C]	Difference (Field - Model_7) [A - B]	Difference (Field - Model_15) [A - C]
1	qlc01	26.7	64	3.313997661	2.353197867	2.797491811	0.960799794	0.51650585
2	qlc02	20.7	31.2	0.67025184	0.517907425	0.635948269	0.152344415	0.034303571
3	qlc03	13.41	15.4	0.13748698	0.083158539	0.102285766	0.054328442	0.035201214
4	qlc04	28.2	71.5	3.502206219	2.947823459	3.479571861	0.55438276	0.022634358
5	qlc05	16.89	28.3	0.452896081	0.416042378	0.429499631	0.036853703	0.023396451
6	qlc06	26.47	56	1.803078527	1.790714672	2.256677148	0.012363855	-0.453598621
7	qlc07	26.5	40.5	1.499873847	0.911274091	1.312327941	0.588599756	0.187545906
8	qlc01	17.95	56.7	1.497397175	1.836982659	1.677658457	-0.339585484	-0.180261282
9	qlc02	19.33	67.1	1.835375021	2.591053649	2.342342794	-0.755678628	-0.506967774
10	qlc03	17.77	46.1	1.240817978	1.196917851	1.156192084	0.043900128	0.084625894
11	qlc04	22.2	75.3	2.191757047	3.274134203	3.12181326	-1.082377155	-0.930056213
12	qlc05	14.9	36.5	0.69945629	0.729646833	0.626695184	-0.030190543	0.072761106
13	qlc06	11.69	26	0.362295241	0.342302309	0.251948694	0.019992932	0.110346547
14	qlc07	10.3	17.3	0.114967671	0.119848546	0.099190784	-0.004880875	0.015776887
15	qlw01	12.3	33	0.415944907	0.586115396	0.425339995	-0.170170489	-0.009395088
16	qlw02	7.5	13	0.031992597	0.042907599	0.041646467	-0.010915002	-0.009653871
17	qlw04	20.1	61.6	1.642854298	2.176691546	2.109706861	-0.533837248	-0.466852562
18	qlw06	15.2	45.5	0.856030951	1.164570624	0.976467816	-0.308539674	-0.120436865
19	qlw07	13.2	27.4	0.367527492	0.386444557	0.315468694	-0.018917065	0.052058798
20	qlw08	12.93	25	0.180214481	0.312187849	0.257610706	-0.131973368	-0.077396225
21	qlw09	12.76	51	0.568257288	1.476670926	1.026368451	-0.908413639	-0.458111163
22	qlw10	20.3	68	2.389827757	2.662196486	2.484933862	-0.272368729	-0.095106105
23	qlw12	14.75	47.2	0.782527525	1.257303247	1.016899519	-0.474775722	-0.234371995
24	qlw13	18.95	56	1.652609099	1.790714672	1.719619289	-0.138105573	-0.06701019
25	qlw14	14.75	43	0.751317963	1.034285785	0.852540606	-0.282967823	-0.101222643
26	qlw15	16.5	23.5	0.301672115	0.269228925	0.290237414	0.03244319	0.011434701
27	qlw16	17.2	32	0.584819662	0.547751328	0.557310719	0.037068334	0.027508943
28	qlw17	21.6	58.8	1.803271761	1.979178722	2.074348718	-0.175906961	-0.271076957
29	qlw18	19.6	57.9	1.626454861	1.917614462	1.869288088	-0.2911596	-0.242833227
30	qlw19	16.3	21	0.214392596	0.203533284	0.229328799	0.010859312	-0.014936203
31	qlw20	13.9	32.6	0.455662626	0.570628583	0.468588794	-0.114965957	-0.012926168
32	qlw21	6	13.3	0.032386261	0.047566929	0.035110867	-0.015180668	-0.002724606
33	qlw22	12.9	33.5	0.44574497	0.605738555	0.459328007	-0.159993585	-0.013583037
34	qlw23	14	42	0.820796601	0.984206754	0.774987335	-0.163410153	0.045809267

35	qlwc24	19.2	61	1.978420807	2.13360521	2.001477076	-0.155184402	-0.023056268
36	qlwc25	24.8	63	3.064508214	2.278844148	2.578321923	0.785664066	0.486186291
37	qlwc26	17.05	72	2.24734961	2.98979397	2.370907948	-0.74244436	-0.123558338
38	qlwc27	20	63.5	2.180611717	2.315876404	2.205614648	-0.135264687	-0.025002931
39	qlwc28	20.7	77.5	2.003747066	3.4707723	3.091559873	-1.467025234	-1.087812806
40	qlwc29	11.5	17.5	0.086447578	0.123958677	0.113113743	-0.037511099	-0.026666165
41	qlwc31	21	72.5	2.967221644	3.032056888	2.817651773	-0.064835244	0.149569871
42	qlwc32	26	74.5	4.300669356	3.20403333	3.482330929	1.096636027	0.818338427
43	qlw01	27.7	91.5	5.88284685	4.854773185	5.195962349	1.028073666	0.686884502
44	qlw02	27.7	59.8	3.263687688	2.048679877	2.591013928	1.215007811	0.67267376
45	qlw03	20.5	47.5	1.661733057	1.274014923	1.386482233	0.387718133	0.275250824
46	qlw04	19.9	28.3	0.600796126	0.416042378	0.505067583	0.184753748	0.095728543
47	qlw05	13.7	19.3	0.219020093	0.163075787	0.163241467	0.055944306	0.055778626
48	qlw06	18.7	35.6	0.882452559	0.691364602	0.744739883	0.191087957	0.137712677
49	qlw08	28.9	68.6	4.19737754	2.710147974	3.319892	1.487229565	0.87748554
				70.78305329	70.83358036	70.84215204	-0.050527069	-0.059098751

From the above table, the difference [A-B] provides difference between the volume measured in the field (actual volume) and the volume predicted by model 7. The figures with negative (-) indicates that the volume has been over-predicted by the model 7 vis-à-vis actual volume of the particular tree. And the figures without negative (-) sign indicates the under prediction of volume by the model 7.

Similarly, the difference [A-C] is the difference between the actual volume and the volume predicted by the model 15. Same explanation is applicable here – the figures with negative sign indicates overprediction of volume by the model, while those figures without (-) are under prediction of volume by the model 15.

Summation of the figures in the difference column results in -0.050527069 m^3 and -0.059098751 m^3 for model 7 and model 15 respectively. These indicate that the model 7 over predicts total volume for 49 trees by 0.050527069 m^3 , while the model 15 over predicts the total volume of 49 trees by 0.059098751 m^3 . Therefore, looking this, one may be inclined to conclude that overall, model 7 predicts slightly better than model 15.

12. Limitations of the model

The model has the following limitations;

1. The modeling has been done based on only 49 sample trees. The model can be further improved by increasing the samples.
2. The diameter for the samples ranges between minimum of 13 cm to 91 cm (over bark). Thus, the model prediction for trees above 91 cm must be done with caution.

13. Conclusion

Unlike our observations on modelling other broadleaf species, for *Quercus lanata*, the model Model 15 (fitted with height as predictor) is found to be the best fit model having the lower AIC and BIC values vis-à-vis model Model 7 (fitted without height as predictor).

This, therefore, leads us to confidently conclude that the best model for *Quercus lanata*, out of 16 models fitted above, is model 15.

However, we consider two best fit models for *Quercus lanata*, since, two models have been fitted with height and without height as predictor. Therefore, the best fit models are;

1. Model 7 – the best fit model for models fitted without height
2. Model 15 – the best fit model for models fitted with height

14. Acknowledgement

We would like to express our heartfelt gratitude to the biomass equation development team led by Mr. Yograj Chettri, Research Officer at UWICER, formerly RDC who collected data (diameter and height) for developing merchantable volume equation for *Quercus lanata*, as part of their field work for biomass equation development exercise. The biomass equation development exercise was part of project implemented by Forest Resources Management Division (FRMD).

Immense gratitude is also due to Professor Timothy Gordon Gregoire, School of Forestry and Environmental Studies (FES), Yale University who has been a guide and mentor as we worked on this assignment.

Thanks are also due to our Director, Mr. Lobzang Dorji and other colleagues working in FRMD for their support and advice.

Had it not been for the immense support, guidance and encouragement that the abovementioned people have generously provided to us, this task would have remained incomplete.

15. References

1. Lee, D., Seo, Y., & Choi, J. (2017). Estimation and validation of stem volume equations for *Pinus densiflora*, *Pinus koraiensis*, and *Larix kaempferi* in South Korea. *Forest Science and Technology*, 13(2), 77-82.
2. Umunay, P., Gregoire, T., & Ashton, M. (2017). Estimating biomass and carbon for *Gilbertiodendron dewevrei* (De Wild) Leonard, a dominant canopy tree of African tropical Rainforest: Implications for policies on carbon sequestration. *Forest Ecology and Management*, 404, 31-44.
3. White, J. C., Coops, N. C., Wulder, M. A., Vastaranta, M., Hilker, T., & Tompalski, P. (2016). Remote sensing technologies for enhancing forest inventories: A review. *Canadian Journal of Remote Sensing*, 42(5), 619-641.
4. Mohammadi, J., Shataee, S., & Babanezhad, M. (2011). Estimation of forest stand volume, tree density and biodiversity using Landsat ETM+ Data, comparison of linear and regression tree analyses. *Procedia Environmental Sciences*, 7, 299-304.
5. Fagan, M., & DeFries, R. (2009). Measurement and Monitoring of the World's Forests. *Resources for the Future*, 129.
6. Feng, Z. K., Yang, B.G., Luo, X., Han, G.S., Guo, X.X., (2008). Experiment of estimating forest stand volume with LiDAR technology. *The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences.*, XXXVII.
7. McRoberts, R. E., & Tomppo, E. O. (2007). Remote sensing support for national forest inventories. *Remote Sensing of Environment*, 110(4), 412-419.
8. Westfall, J. A., & Patterson, P. L. (2007). Measurement variability error for estimates of volume change. *Canadian Journal of Forest Research*, 37(11), 2201-2210.
9. Lu, D. (2006). The potential and challenge of remote sensing-based biomass estimation. *International Journal of Remote Sensing*, 27(7), 1297-1328.
10. Sadiq, R. A. (2006). A new approach to log volume estimation. *Southern Journal of Applied Forestry*, 30(1), 30-39.
11. Hyypä, J., Mielonen, T., Hyypä, H., Maltamo, M., Yu, X., Honkavaara, E., & Kaartinen, H. (2005). Using individual tree crown approach for forest volume extraction with aerial images and laser point clouds.
12. Patterson, D. W., & Doruska, P. F. (2004). A new and improved modification to Smalian's equation for butt logs. *Forest Products Journal*, 54(4), 69.
13. Eerikäinen, K. (2001). Stem volume models with random coefficients for *Pinus kesiya* in Tanzania, Zambia, and Zimbabwe. *Canadian Journal of Forest Research*, 31(5), 879-888.
14. Gregoire, T. G., & Schabenberger, O. (1996). Nonlinear mixed-effects modeling of cumulative bole volume with spatially correlated within-tree data. *Journal of Agricultural, Biological, and Environmental Statistics*, 107-119.
15. Bi, H. (1994). Volume equations for six Eucalyptus species on the south-east tablelands of New South Wales: Research Division State Forests of New South Wales.
16. Laumans, P. (1994). Height-diameter functions from PIS for country-level site classification and local volume table selection. Thimphu.
17. Biging, G. S. (1988). Estimating the accuracy of volume equations using taper equations of stem profile. *Canadian Journal of Forest Research*, 18(8), 1002-1007.
18. Reed, D. D., & Byrne, J. C. (1985). A simple, variable form volume estimation system. *The Forestry Chronicle*, 61(2), 87-90.
19. Avery, T.E., and Burkhart, H.E. (1983). *Forest Measurements*, Third Edition. McGraw-Hill, Inc.
20. Sadiq, R. A., & Smith, V. G. (1983). Estimation of individual tree volumes with age and diameter. *Canadian Journal of Forest Research*, 13(1), 32-39.
21. Cochran, P. (1982). Estimating wood volumes for Douglas-fir and white fir from outside bark measurements. *Forest Science*, 28(1), 172-174.

22. Bredenkamp, B. (1982). Volume regression equations for *Eucalyptus grandis* on the coastal plain of Zululand. *South African Forestry Journal*, 122(1), 66-69.
23. Cao, Q. v., & Burkhart, H. E. (1980). Cubic-foot volume of loblolly pine to any height limit. *Southern Journal of Applied Forestry*, 4(4), 166-168.
24. Cao, Q. V., Burkhart, H. E., & Max, T. A. (1980). Evaluation of two methods for cubic-volume prediction of loblolly pine to any merchantable limit. *Forest Science*, 26(1), 71-80.
25. Goulding, C. (1979). Cubic spline curves and calculation of volume of sectionally measured trees. *NZJ For. Sci*, 9(1), 89-99.
26. Burkhart, H. E. (1977). Cubic-foot volume of loblolly pine to any merchantable top limit. *Southern Journal of Applied Forestry*, 1(2), 7-9.
27. Heger, L. (1965). A trial of Hohenadl's method of stem form and stem volume estimation. *The Forestry Chronicle*, 41(4), 466-475.

16. Annexure – Dataset for *Quercus lanata*

SN	Tree_ID	Height.m	DBH.cm	Volume.m3	BA.m2	BAH.m3	DBH2H.m3
1	qle01	26.7	64	3.313997661	0.321699088	8.589365642	10.93632
2	qle02	20.7	31.2	0.67025184	0.076453799	1.582593636	2.0150208
3	qle03	13.41	15.4	0.13748698	0.018626503	0.249781403	0.31803156
4	qle04	28.2	71.5	3.502206219	0.401515176	11.32272797	14.416545
5	qle05	16.89	28.3	0.452896081	0.062901754	1.062410617	1.35270321
6	qle06	26.47	56	1.803078527	0.246300864	6.519583871	8.300992
7	qle07	26.5	40.5	1.499873847	0.128824934	3.413860744	4.3466625
8	qlec01	17.95	56.7	1.497397175	0.25249687	4.532318819	5.77072755
9	qlec02	19.33	67.1	1.835375021	0.353618454	6.835444725	8.70315853
10	qlec03	17.77	46.1	1.240817978	0.166913603	2.966054727	3.77649817
11	qlec04	22.2	75.3	2.191757047	0.445327827	9.886277765	12.5875998
12	qlec05	14.9	36.5	0.69945629	0.10463467	1.559056588	1.9850525
13	qlec06	11.69	26	0.362295241	0.053092916	0.620656186	0.790244
14	qlec07	10.3	17.3	0.114967671	0.023506182	0.242113671	0.3082687
15	qlwc01	12.3	33	0.415944907	0.08552986	1.052017278	1.33947
16	qlwc02	7.5	13	0.031992597	0.013273229	0.099549217	0.12675
17	qlwc04	20.1	61.6	1.642854298	0.298024045	5.990283314	7.6270656
18	qlwc06	15.2	45.5	0.856030951	0.162597055	2.471475233	3.14678
19	qlwc07	13.2	27.4	0.367527492	0.058964553	0.778332093	0.9910032
20	qlwc08	12.93	25	0.180214481	0.049087385	0.634699891	0.808125
21	qlw09	12.76	51	0.568257288	0.204282062	2.606639115	3.318876
22	qlwc10	20.3	68	2.389827757	0.363168111	7.372312648	9.38672
23	qlwc12	14.75	47.2	0.782527525	0.174974144	2.58086863	3.286064
24	qlwc13	18.95	56	1.652609099	0.246300864	4.667401374	5.94272
25	qlwc14	14.75	43	0.751317963	0.14522012	2.141996776	2.727275
26	qlwc15	16.5	23.5	0.301672115	0.043373614	0.715664624	0.9112125
27	qlwc16	17.2	32	0.584819662	0.080424772	1.383306077	1.76128
28	qlwc17	21.6	58.8	1.803271761	0.271546703	5.865408776	7.4680704
29	qlwc18	19.6	57.9	1.626454861	0.263297666	5.160634248	6.5707236
30	qlwc19	16.3	21	0.214392596	0.034636059	0.564567762	0.71883
31	qlwc20	13.9	32.6	0.455662626	0.083468975	1.160218755	1.4772364
32	qlwc21	6	13.3	0.032386261	0.013892908	0.083357449	0.106134
33	qlwc22	12.9	33.5	0.44574497	0.088141309	1.137022885	1.4477025
34	qlwc23	14	42	0.820796601	0.138544236	1.939619304	2.4696
35	qlwc24	19.2	61	1.978420807	0.292246657	5.611135807	7.14432
36	qlwc25	24.8	63	3.064508214	0.311724531	7.73076837	9.84312
37	qlwc26	17.05	72	2.24734961	0.407150408	6.941914455	8.83872
38	qlwc27	20	63.5	2.180611717	0.316692174	6.333843489	8.0645
39	qlwc28	20.7	77.5	2.003747066	0.471729772	9.764806278	12.4329375

Merchantable_volume_equation_Quercus lanata: 35

40	qlwc29	11.5	17.5	0.086447578	0.024052819	0.276607416	0.3521875
41	qlwc31	21	72.5	2.967221644	0.41282491	8.669323102	11.038125
42	qlwc32	26	74.5	4.300669356	0.435915616	11.33380601	14.43065
43	qlw01	27.7	91.5	5.88284685	0.657554977	18.21427287	23.1911325
44	qlw02	27.7	59.8	3.263687688	0.280861525	7.779864238	9.9056308
45	qlw03	20.5	47.5	1.661733057	0.177205461	3.632711943	4.6253125
46	qlw04	19.9	28.3	0.600796126	0.062901754	1.251744895	1.5937711
47	qlw05	13.7	19.3	0.219020093	0.029255296	0.400797558	0.5103113
48	qlw06	18.7	35.6	0.882452559	0.099538222	1.861364745	2.3699632
49	qlw08	28.9	68.6	4.19737754	0.369605234	10.68159127	13.6002244