



དབལ་ཕྱན་འབྲུག་གཞུང་། སོ་ནམ་དང་ནགས་  
ཚལ་ལྷན་ཁག། རགས་ཚལ་དང་སྐྱིང་ཀ་ཞབས་  
རྟོག་ལས་ཁུངས།



Royal Government of Bhutan  
Ministry of Agriculture and Forests  
Department of Forests and Park Services

---

# SPECIES SPECIFIC VOLUME EQUATION TO ESTIMATE MERCHANTABLE VOLUME

*Juniperus recurva*

---

Species specific volume equation to  
estimate merchantable volume

*Juniperus recurva*

December, 2018

## Table of Contents

1. Summary.....	1
2. Introduction.....	2
3. Volume Calculation.....	3
4. The Dataset used for modeling volume of <i>Juniperus recurva</i> .....	4
4.1 Summary descriptive statistics of <i>Juniperus recurva</i> dataset.....	4
5. Fitting the models.....	5
6. Summary Plots.....	6
7. Models and results.....	7
7.1 Model 1 - Volume with diameter at breast height (DBH) as predictor .....	7
7.2 Model 2 - Volume with diameter at breast height (DBH) as predictor, with varFixed .....	8
7.3 Model 3- Volume with diameter at breast height (DBH) as predictor, with varPower.....	9
7.4 Model 4 - Volume with diameter at breast height (DBH) as predictor, with varConstPower ...	10
7.5 Model 5 - Volume with basal area (BA) as predictor.....	11
7.6 Model 6 - Volume with basal area (BA) as predictor, with varFixed.....	12
7.7 Model 7 Volume with basal area (BA) as predictor, with varPower.....	13
7.8 Model 8 – Volume with basal area (BA) as predictor, with varConstPower.....	14
7.9 Model 9 – Volume with square of diameter at breast height * height (DBH2H) as predictor..	15
7.10 Model 10 – Volume with square of diameter at breast height * height (DBH2H) as predictor, with varFixed.....	16
7.11 Model 11– Volume with square of diameter at breast height * height (DBH2H) as predictor, with varPower.....	17
7.12 Model 12 –Volume with square of diameter at breast height * height (DBH2H) as predictor, with varConstPower .....	18
7.13 Model 13 – Volume with basal area * height (BAH) as predictor.....	19
7.14 Model 14 – Volume with basal area * height (BAH) as predictor, with varFixed .....	20
7.15 Model 15– Volume with basal area * height (BAH) as predictor, with varPower.....	21
7.16 Model 16 – Volume with basal area * height (BAH) as predictor, with varConstPower .....	22
1. Model evaluation using AIC and BIC values.....	23
8. Selected Models .....	25
9. Demonstration of use of the selected best fit model.....	25
10. Model Performance.....	28
11. Limitations of the model.....	29
12. Conclusion.....	30
13. Acknowledgement.....	31
14. References.....	32
15. Annexure – Dataset for <i>Juniperus recurva</i> .....	34

## 1. Summary

The volume equation developed in this study will predict the merchantable volume of *Juniperus recurva*. The merchantability standard adopted for this study are the trees above 10 cm in diameter at breast height (dbh) and top diameter measured up to 10 cm over bark have been considered for volume calculation.

A total of 16 models were fitted. First 4 models were fitted with volume as a function of diameter at breast height (DBH), while models 5 – 8 were fitted with basal area (BA) as the predictor variable. With product of squared diameter at breast height and height (DBH2H) as predictor variable, 4 models, namely the models 9 – 12 were fitted. The last four models, 13 -16 were fitted with product of basal area and height (BAH) as the predictor.

The initial plots of response (volume) variable and predictor (DBH, BA, DBH2H and BAH) variables clearly indicated presence of heteroscedasticity, which has been modeled using variance functions (varFixed, varPower and varConstPower) in gls () function of nlme package.

Of the sixteen, two models viz model 7 (fitted with BA as predictor) and model 15 (fitted with BAH as predictor) with lowest values of AIC and BIC have been selected as the best fit models for *Juniperus recurva*. The model 7 had AIC and BIC values of -15 and -6 respectively, while the model 15 had AIC and BIC values of -43 and -34 respectively. Lower the AIC and BIC values, better the fit of the model.

The performance of the selected models was assessed by comparing the actual volume with the volumes predicted by two selected models for each tree. From the assessment, the model 15 which uses height outperforms the model 7.

## 2. Introduction

The volume equations, developed during pre-investment survey (PIS) carried out between 1974-81 predict total tree volume, and not the merchantable volume of trees. The recent change of policy of the Department of Forests and Park Services to allot timber for rural house construction in the form of log volume instead of allotting by number of trees as was once practiced, has necessitated development of merchantable log volume equation.

Therefore, standards of merchantability adopted for this study to develop merchantable log volume equation are all trees above 10 cm diameter at breast height (dbh) and the sections up to 10 cm top diameter over the bark have been considered for volume calculation.

As was done for PIS exercise to develop volume equation, this study ignores/does not consider the volume of foliage and branches for the purpose of calculating the merchantable volume. This decision stems from the objective, which is to estimate merchantable log volume. Moreover, branches are rarely used as timber (at least in Bhutan) and are mostly used for firewood.

The sample trees for this study have been felled as part of biomass equation development field work. The data protocol for biomass equation development required collecting a minimum of 8 trees each from four regions of Bhutan namely, eastern, eastern central, western and western central. Therefore, 49 trees in total have been felled for *Juniperus recurva* from four regions namely; eastern, east-central, western-central and western regions.

The trees were felled at 0.3 m height from the ground at which the diameter was measured and recorded. After felling diameter was measured at 0.7 m from 0.3 m height (essentially making 1 m height, i.e  $0.3\text{ m} + 0.7\text{ m} = 1\text{ m}$ ). Thereafter, at every meter length, the diameter was measured and recorded, thus making many 1 m length sections of log. As mentioned above the smallest top diameter considered for merchantable log volume calculation was up to 10 cm diameter over bark. Top sections below 10 cm diameter have been discarded.

### 3. Volume Calculation

Trees after felling are converted into different sizes of sections depending on the requirement and demand. Sections with length of 8 or more feet long are called logs and shorter ones are called sticks or bolts (Avery and Burkhardt, 1994). The scaling or measuring the volume of the section is done by multiplying the length with the cross-sectional area of the section. Although they rarely form true circles, they are assumed so for the purpose of calculating cross sectional area in meter square, which is

$$\text{Cross sectional area (A)} = A = \pi r^2 = \frac{\pi D^2}{4 \times 10000} \quad (1)$$

Where **r** is radius in meters and **D** is diameter at breast height in centimeters.

From the ground level to 0.3 m height (height at which sample tree has been cut) is section I, while 0.3 m to 0.7 m is section II. The subsequent sections of 1 m length each are numbered III, IV and so on. The last section is the terminal section, whose length is equal to or less than 1 m. This study has not included the branch volumes in merchantable volume calculation.

The diameter at zero height (ground level) for stump wasn't measured in the field (for those sample trees for which volume data was collected during biomass equation development field work) and therefore, calculated based on diameter reading at 0.3 m height. Therefore, diameter at zero height was calculated as 10% more than diameter at 0.3 m height, which is;

$$D_{(\text{ground})} = D_{(0.3 \text{ m})} + 10\% * D_{(0.3 \text{ m})} \quad (2)$$

Where;

$D_{(\text{ground})}$  is diameter in centimeter of tree at ground level  
 $D_{(0.3 \text{ m})}$  is diameter in centimeter of tree at 0.3 m height

For instance, if  $D_{(0.3 \text{ m})}$  was 70 cm, the  $D_{(\text{ground})}$  is calculated as;

$$\begin{aligned} D_{(\text{ground})} &= 70 \text{ cm} + 10\% \text{ of } 70 \text{ cm} \\ &= 70 + 7 \\ &= 77 \text{ cm} \end{aligned}$$

The most commonly used formulae for calculating volume are the Huber, Newton and Smalian's formulae (Sadiq, 2006, and Goulding, 1979). Of the three commonly used volume calculation approaches or formulae, the Smalian's formula has been used to calculate volume (in m<sup>3</sup>) for this study, as under;

$$\text{Section volume (V}_s) = \frac{A+a}{2} * L \quad (3)$$

Where  $A$  = Cross sectional area in m<sup>2</sup> at large end of the section  
 $a$  = Cross sectional area in m<sup>2</sup> at small end of the section  
 $L$  = Length of the section in meter

Smalian's formula is the easiest and least expensive to apply and therefore applied to get volume for each section of the sample trees. However, for the terminal section, the following formula was used to calculate the volume;

$$\text{Terminal section volume (V}_t) = \frac{A}{3} * L \quad (4)$$

The volume for sections and terminal section for individual trees were then summed to obtain the total volume for each individual sample tree, which is;

$$\text{Volume of tree (V)} = \sum_{s=1}^n V_s + V_t \quad (5)$$

After obtaining individual tree volume (Volume.m3), it was then tabulated against the variables - height in meter (Height.m) and the diameter at breast height in centimeter (DBH.cm).

#### 4. The Dataset used for modeling volume of *Juniperus recurva*

A total of 49 sample trees have been felled and collected data for modeling *Juniperus recurva* from four regions – eastern, eastern central, western central and western, as defined in the protocol for biomass equation development field work. The summary of dataset is presented below, while the detailed dataset is presented as an annexure to this document.

##### 4.1 Summary descriptive statistics of *Juniperus recurva* dataset

```
> summary(jr)
```

<b>Tree.ID</b>	<b>Height.m</b>	<b>DBH.cm</b>	<b>Volume.m3</b>
jre01 : 1	Min. : 7.38	Min. :10.30	Min. :0.03332
jre02 : 1	1st Qu.:15.78	1st Qu.:26.80	1st Qu.:0.47365
jre03 : 1	Median :19.90	Median :39.00	Median :1.12943
jre04 : 1	Mean :19.04	Mean :40.73	Mean :1.56934
jre05 : 1	3rd Qu.:22.58	3rd Qu.:54.00	3rd Qu.:2.07326
jre06 : 1	Max. :32.40	Max. :85.00	Max. :8.19720

<b>BA.m2</b>	<b>BAH.m3</b>	<b>DBH2H.m3</b>
Min. :0.008332	Min. : 0.07457	Min. : 0.09495
1st Qu.:0.056410	1st Qu.: 0.93077	1st Qu.: 1.18510
Median :0.119459	Median : 2.59226	Median : 3.30057
Mean :0.158483	Mean : 3.64104	Mean : 4.63591
3rd Qu.:0.229022	3rd Qu.: 4.90107	3rd Qu.: 6.24024
Max. :0.567450	Max. : 18.38539	Max. : 23.40900

## 5. Fitting the models

The models have been fitted in R, which is a robust statistical computing environment. It is a powerful tool which provides wide range of statistical and graphical options to explore, calculate and manage data besides modelling. It is very powerful and widely used statistical tool which is free and allows user to customize the scripts depending on desired output, which is not possible in many of the statistical softwares.

After reading in the excel files into R, we created other variables namely; basal area in square meter (BA.m2), basal area in meter times height in meter (BAH.m3) and square of the diameter in meter times height in meter (DBH2H.m3). The height in meter (Height.m) and diameter in centimeter (DBH.cm) were measured and recorded in the field.

Prior to fitting models, we explored and examined each set of data by preparing descriptive summaries that provided mean, median and range of dependent/response and independent/predictor variables. Then we plotted scatter graphs which provided sense of relationship between the dependent/response (volume) and independent/predictor variables (namely DBH.cm, BA.m2, DBH2H.m3 and BAH.m3). These graphs showed curvilinear relationship between response and predictor variables. The scatter plots also clearly revealed the presence of phenomenon, referred in statistical parlance, as heteroscedasticity, which is the increase in variation in response (volume) variable with increase in value of the predictor variables.

Therefore, we fitted the models using the `gls ()` function of the `nlme` package of R, because the `gls ()` function has the capability to model heteroscedasticity. We didn't transform the variables, mainly response variable, because transformation makes it difficult to directly interpret the relationship between response and predictor variables; and secondly to compare the AIC and BIC values among the different models, the response variables need to be identical.

The models were fitted with volume as a function of four variables;

- 1) DBH.cm,
- 2) BA.m2,
- 3) DBH2H.m3 and
- 4) BAH.m3.

For each of the variable, we fitted one simple `gls ()` function, which can be written in the following form;

$$Y = \beta_0 + \beta_1 X + \epsilon, \quad (6)$$

Where  $Y = \text{Volume (V)}$  and  $X = \text{predictor variable}$

And then fitted 3 models with restricted natural cubic spline functions. The restricted natural cubic spline function enables better tracking of curvilinear relationship between response and predictor variables. These models introduce an additional predictor variable as part of a 3 knot-cubic spline. They take the following forms;

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon, \quad (7)$$

Where

$Y = \text{Response variable, volume (V)}$

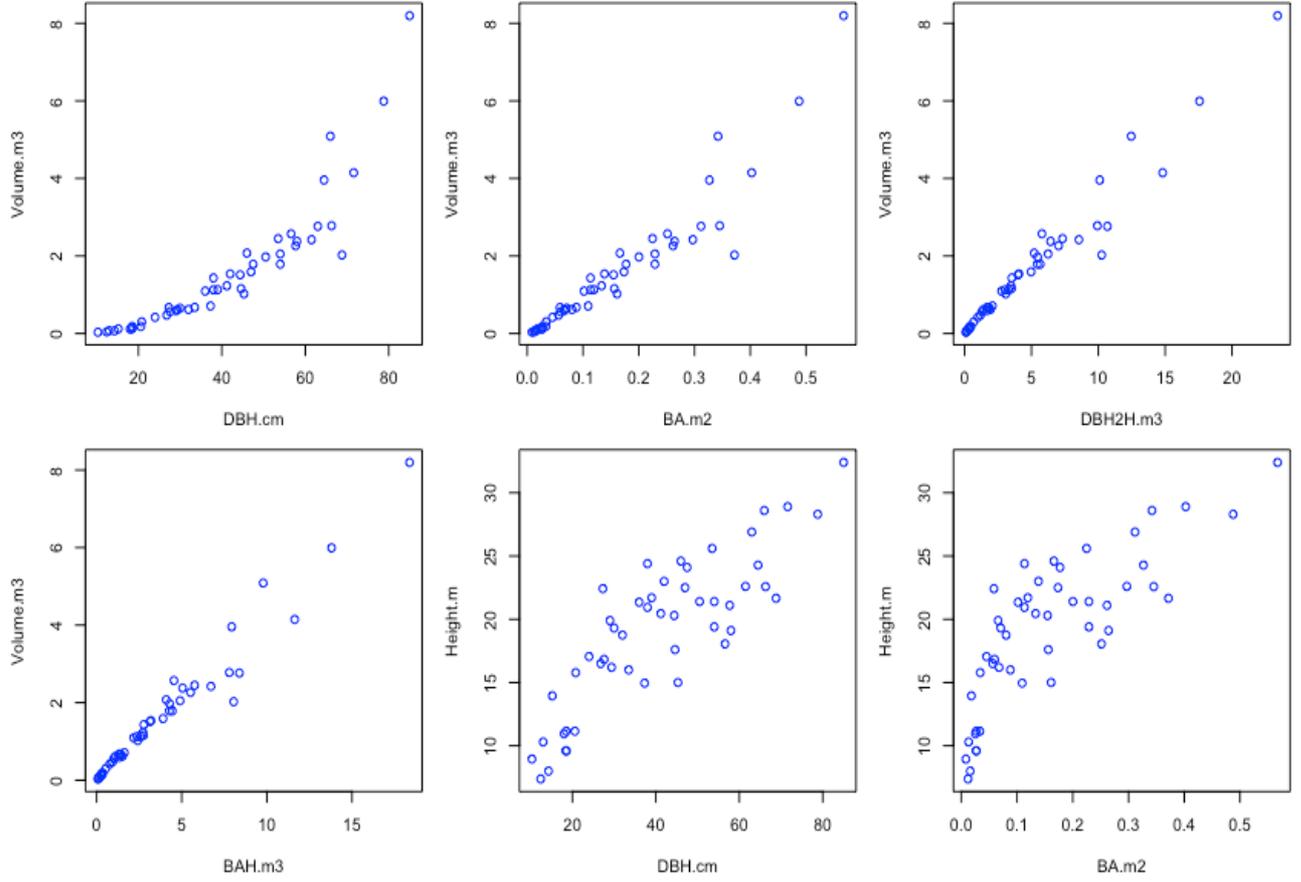
$X_1 = \text{Predictor variable}$

$X_2 = g(X_1)$

And  $g(X_1)$  is the spline transformation of  $X_1$  predictor variable

6. Summary Plots

*Juniperus recurva* (N = 49)



## 7. Models and results

### 7.1 Model 1 - Volume with diameter at breast height (DBH) as predictor

```
> jr.m1 <- gls(Volume.m3 ~ DBH.cm)
> summary(jr.m1)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ DBH.cm

Data: NULL

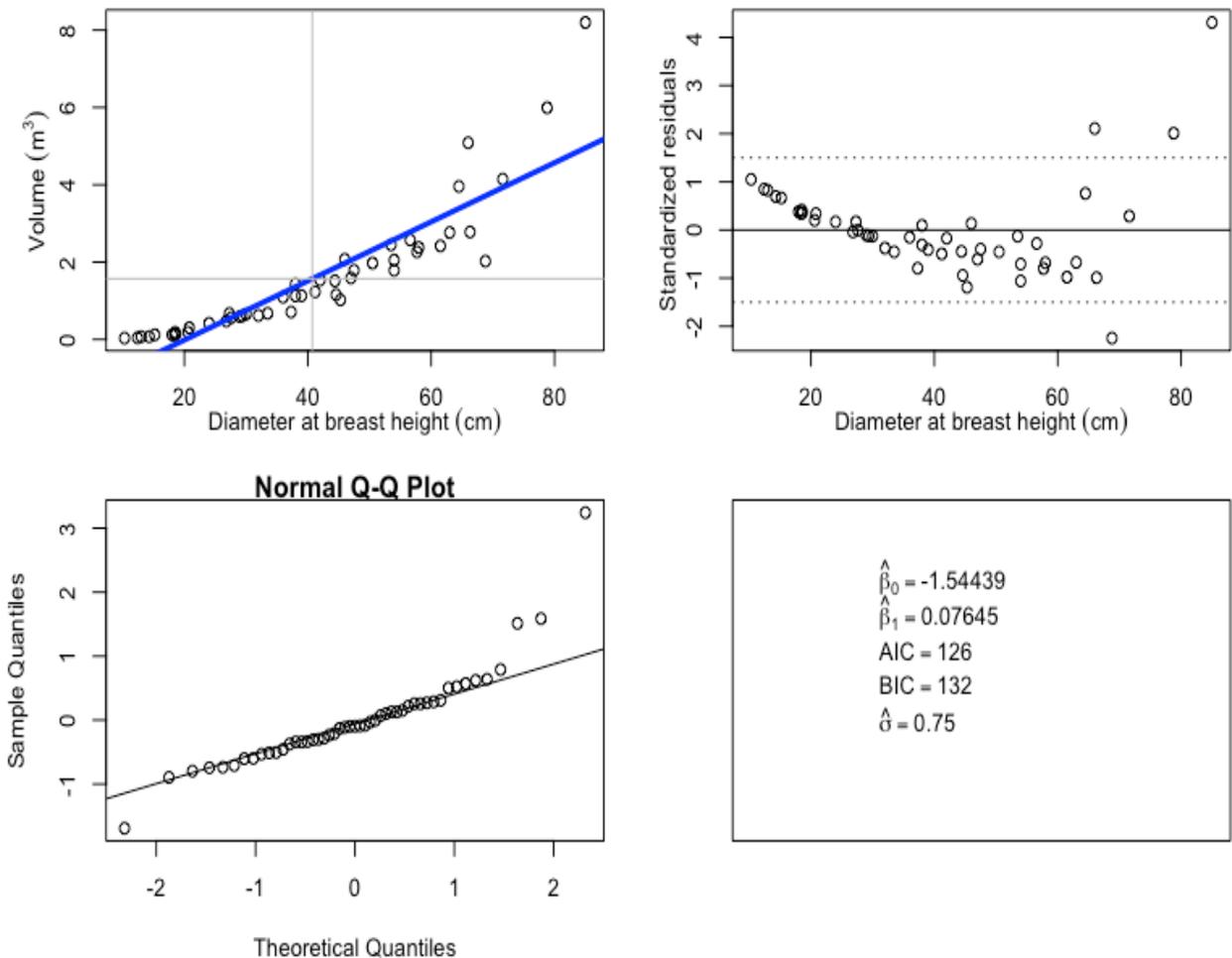
	AIC	BIC	logLik
	126.367	131.9175	-60.18352

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-1.544388	0.25503790	-6.055526	0
DBH.cm	0.076447	0.00567751	13.464870	0

#### Plot of model 1

### J\_recurva:Model 1: (Volume ~ dbh)



## 7.2 Model 2 - Volume with diameter at breast height (DBH) as predictor, with varFixed

```
> jr.m2 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints,
  na.action=na.omit, weights = varFixed(~DBH.cm))
> summary(jr.m2)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ DBH.cm + DBH.cm.splinepoints

Data: NULL

AIC BIC logLik

96.73059 104.0452 -44.3653

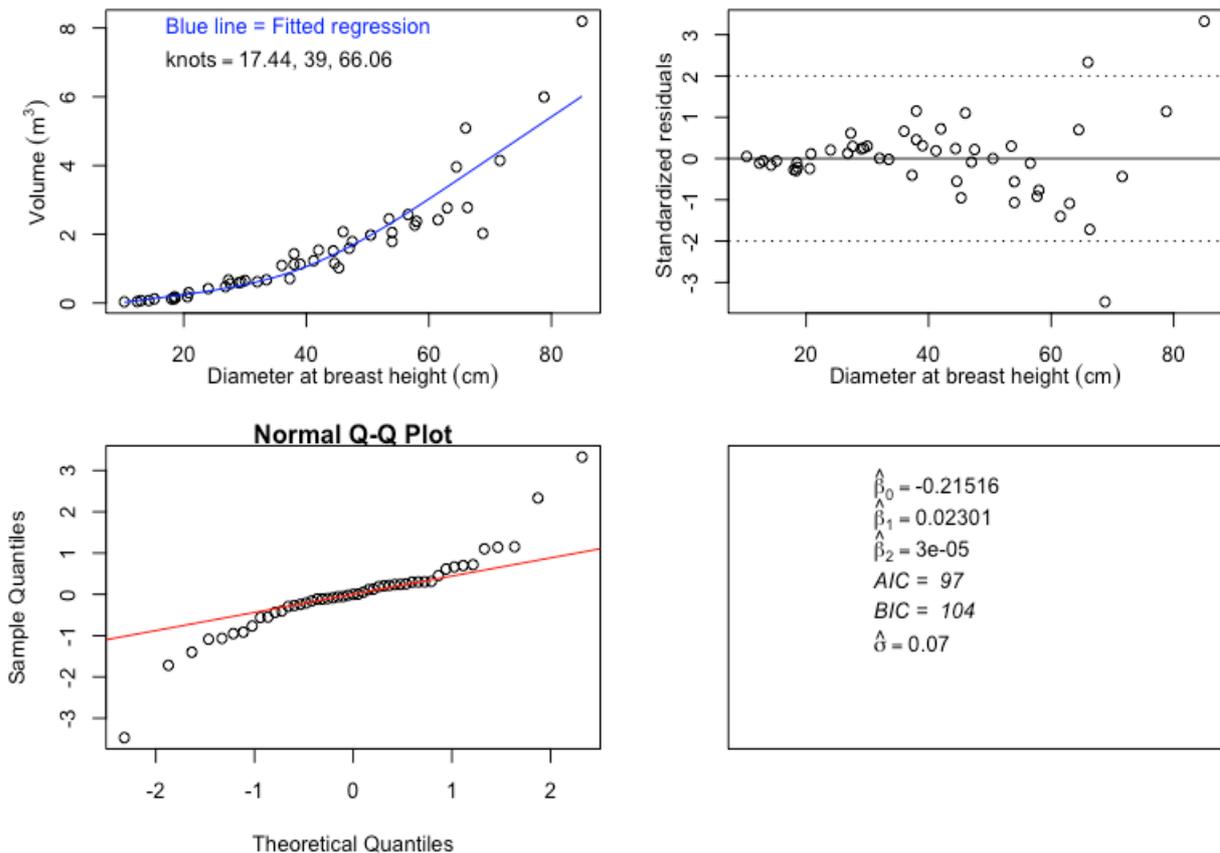
Variance function:

Structure: fixed weights

Formula: ~DBH.cm

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.21516177	0.17393640	-1.237014	0.2224
DBH.cm	0.02301150	0.00720362	3.194435	0.0025
DBH.cm.splinepoints	0.00003077	0.00000477	6.454396	0.0000

**Plot of Model 2****J\_recurva:Model 2 : (Volume ~ dbh), Cubic spline with varFixed**

7.3 Model 3- Volume with diameter at breast height (DBH) as predictor, with varPower

```
> jr.m3 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints,
  na.action=na.omit, weights = varPower(form =
  ~DBH.cm))
> summary(jr.m3)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ DBH.cm + DBH.cm.splinepoints

Data: NULL

	AIC	BIC	logLik
	41.97985	51.12306	-15.98993

Variance function:

Structure: Power of variance covariate

Formula: ~DBH.cm

Parameter estimates:

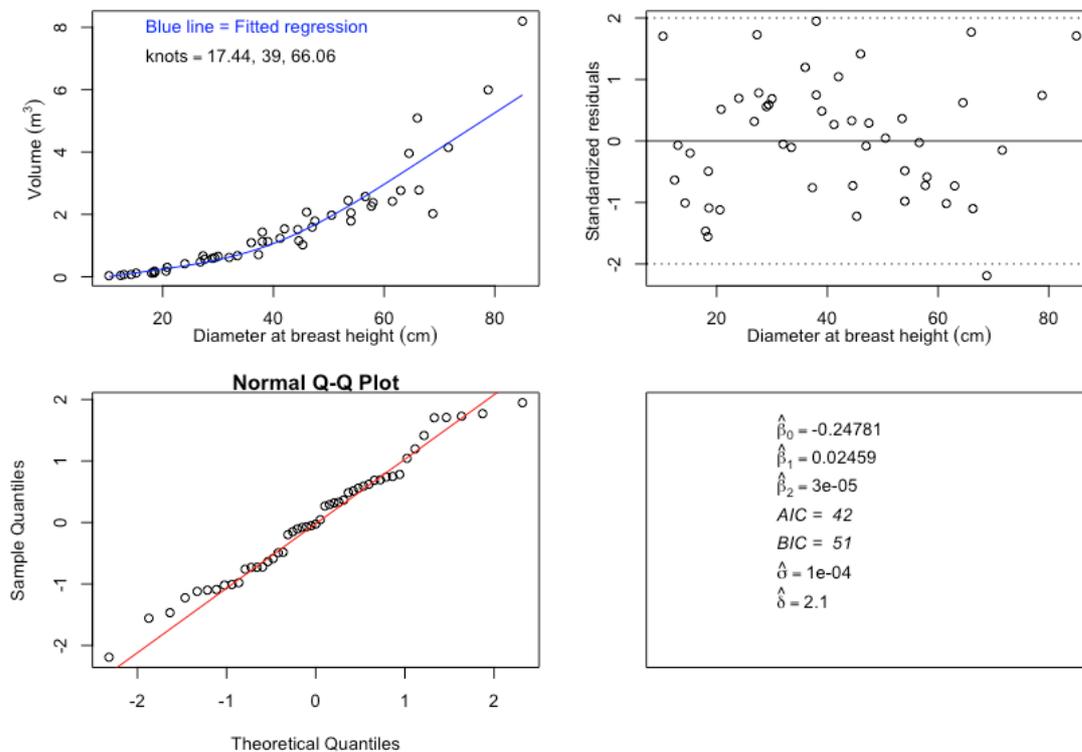
power
2.104034

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.24781231	0.031342890	-7.906492	0
DBH.cm	0.02459423	0.002141841	11.482749	0
DBH.cm.splinepoints	0.00002871	0.000003240	8.862897	0

### Plot of Model 3

#### J\_recurva:Model 3: (Volume ~ dbh), Cubic spline with varPower



7.4 Model 4- Volume with diameter at breast height (DBH) as predictor, with varConstPower

```
> jr.m4 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints,
  na.action=na.omit, weights = varConstPower(form =
  ~DBH.cm))
> summary(jr.m4)
```

Generalized least squares fit by REML  
 Model: Volume.m3 ~ DBH.cm + DBH.cm.splinepoints  
 Data: NULL  
 AIC BIC logLik  
 38.36521 49.33706 -13.18261

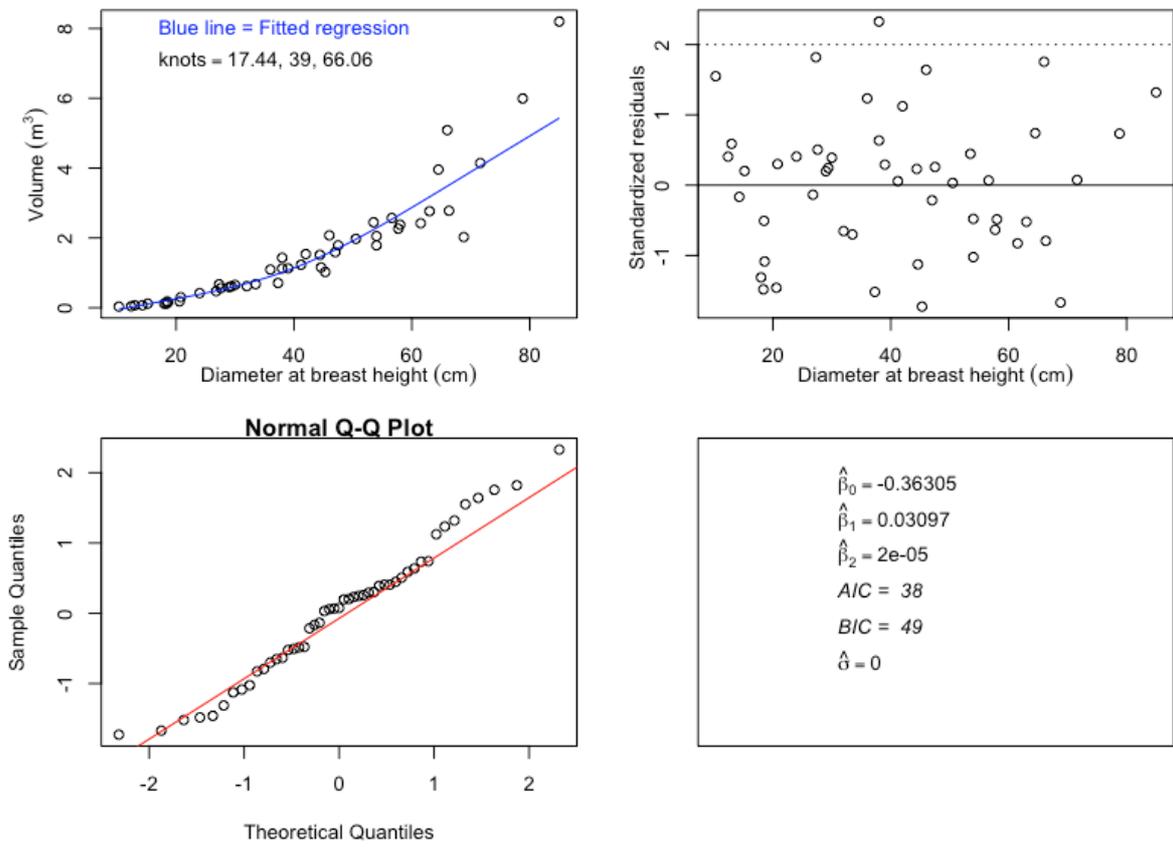
Variance function:  
 Structure: Constant plus power of variance covariate  
 Formula: ~DBH.cm  
 Parameter estimates:  
 const power  
 86616.621818 3.402324

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.3630458	0.04785934	-7.585684	0
DBH.cm	0.0309719	0.00245628	12.609273	0
DBH.cm.splinepoints	0.0000228	0.00000353	6.458053	0

**Plot of Model 4**

J\_recurva:Model 4: (Volume ~ dbh), Cubic spline with varConstPower



7.5 Model 5 - Volume with basal area (BA) as predictor

```
> jr.m5 <- gls(Volume.m3 ~ BA.m2)
> summary(jr.m5)
```

Generalized least squares fit by REML

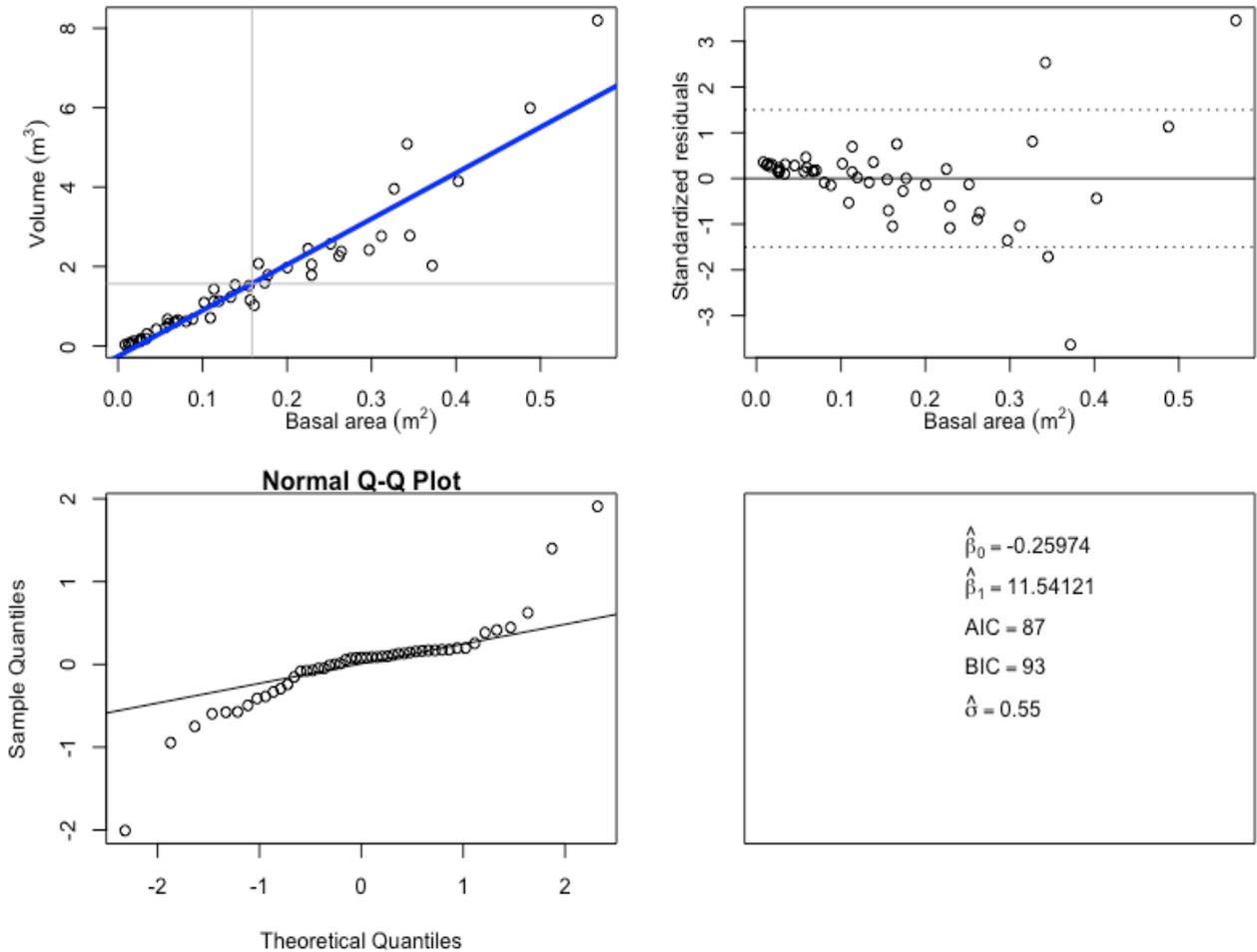
```
Model: Volume.m3 ~ BA.m2
Data: NULL
      AIC      BIC    logLik
87.24195 92.79239 -40.62097
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.259743	0.1227430	-2.116151	0.0397
BA.m2	11.541205	0.5936323	19.441675	0.0000

Plot of Model 5

J\_recurva:Model 5: (Volume ~ BA)



7.6 Model 6 - Volume with basal area (BA) as predictor, with varFixed

```
> jr.m6<- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
              na.action=na.omit, weights = varFixed(~BA.m2))
> summary(jr.m6)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ BA.m2 + BA.m2.splinepoints  
 Data: NULL  
 AIC BIC logLik  
 21.63611 28.95068 -6.818057

Variance function:

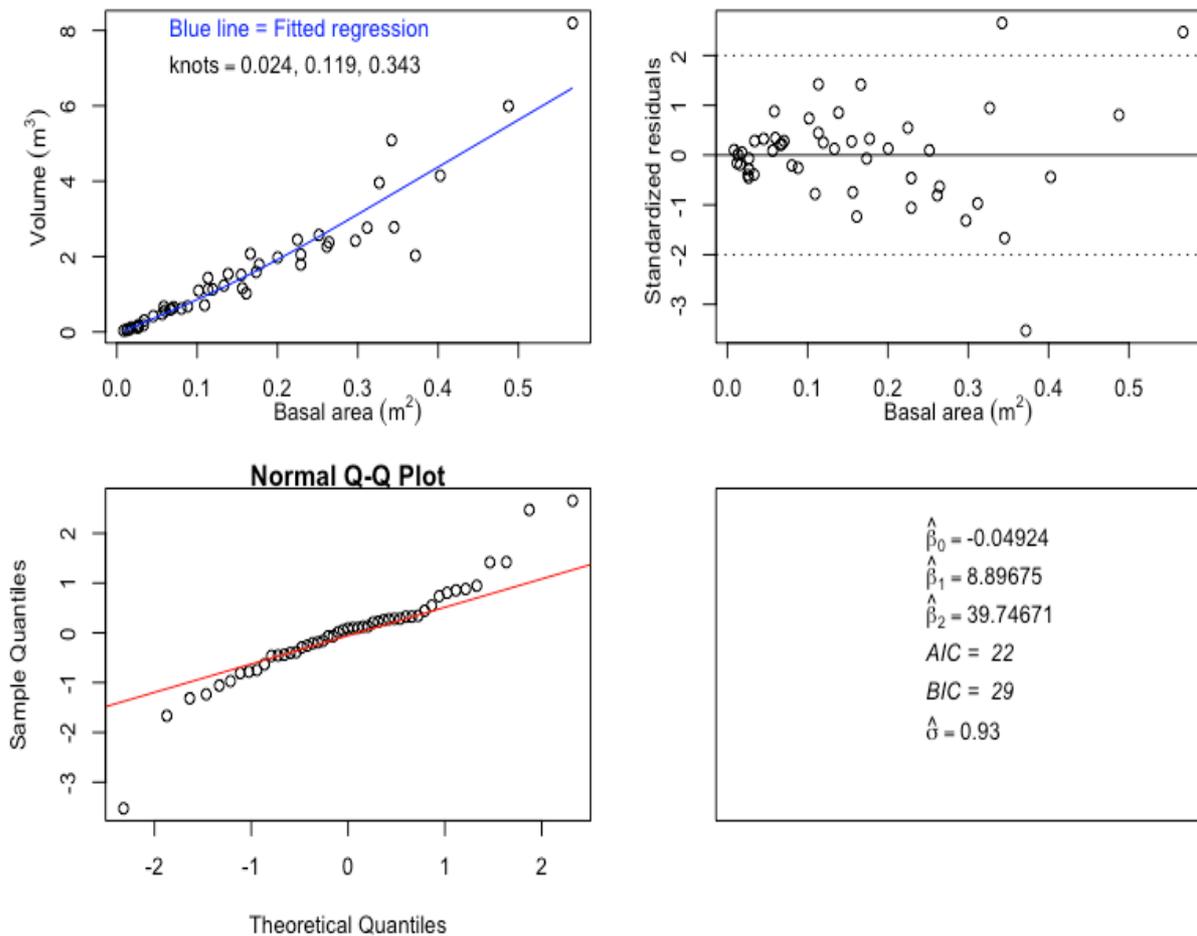
Structure: fixed weights  
 Formula: ~BA.m2

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.04924	0.046424	-1.060575	0.2944
BA.m2	8.89675	0.915640	9.716428	0.0000
BA.m2.splinepoints	39.74671	19.690581	2.018564	0.0494

Plot of Model 6

J\_recurva:Model 6: (Volume ~ BA), Cubic spline with varFixed



7.7 Model 7 Volume with basal area (BA) as predictor, with varPower

```
> jr.m7 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
               na.action=na.omit, weights = varPower(form = ~BA.m2))
> summary(jr.m7)
```

Generalized least squares fit by REML  
 Model: Volume.m3 ~ BA.m2 + BA.m2.splinepoints  
 Data: NULL  
           AIC          BIC     logLik  
-14.87417 -5.730962 12.43708

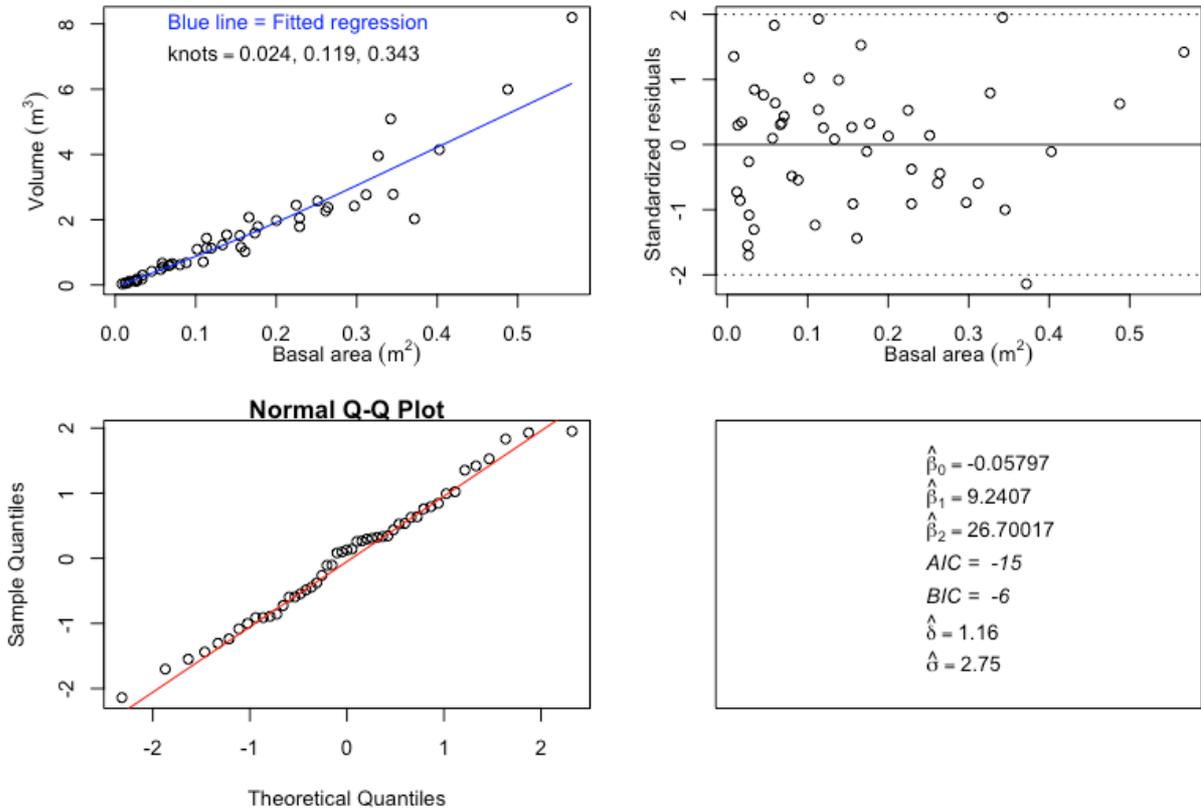
Variance function:  
 Structure: Power of variance covariate  
 Formula: ~BA.m2  
 Parameter estimates:  
   power  
1.161971

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.057974	0.010241	-5.660750	0.0000
BA.m2	9.240696	0.496100	18.626693	0.0000
BA.m2.splinepoints	26.700167	17.881021	1.493213	0.1422

Plot of Model 7

J\_recurva:Model 7: (Volume ~ BA), Cubic spline with varPower



7.8 Model 8 – Volume with basal area (BA) as predictor, with varConstPower

```
> jr.m8 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
  na.action=na.omit, weights = varConstPower(form =
  ~BA.m2))
> summary(jr.m8)
```

Generalized least squares fit by REML  
 Model: Volume.m3 ~ BA.m2 + BA.m2.splinepoints  
 Data: NULL  
 AIC BIC logLik  
 -14.11909 -3.147245 13.05955

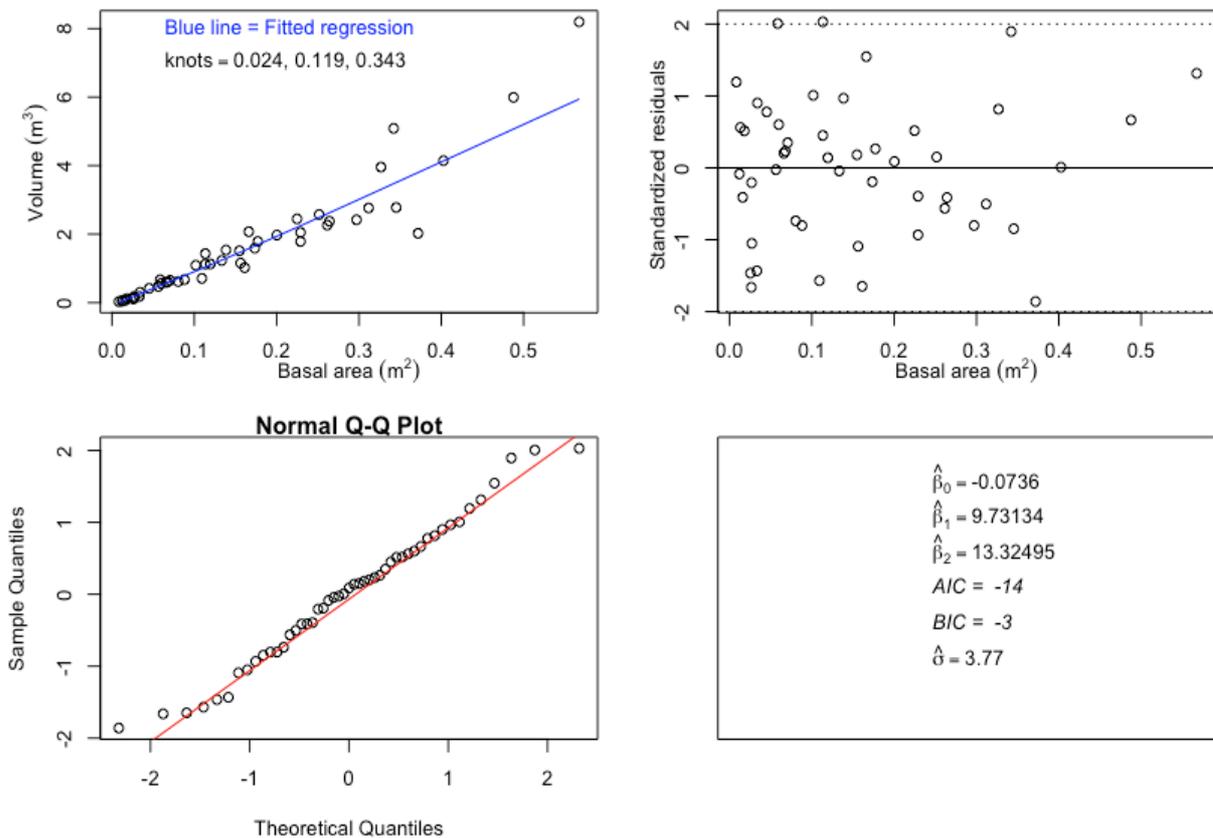
Variance function:  
 Structure: Constant plus power of variance covariate  
 Formula: ~BA.m2  
 Parameter estimates:  
 const power  
 0.004553356 1.407289682

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.073602	0.014522	-5.068376	0.0000
BA.m2	9.731344	0.504686	19.281961	0.0000
BA.m2.splinepoints	13.324952	19.031542	0.700151	0.4874

Plot of Model 8

J\_recurva:Model 8: (Volume ~ BA), Cubic spline with varConstPower



7.9 Model 9 – Volume with square of diameter at breast height \* height (DBH2H) as predictor

```
> jr.m9 <- gls(Volume.m3 ~ DBH2H.m3)
> summary(jr.m9)
```

Generalized least squares fit by REML

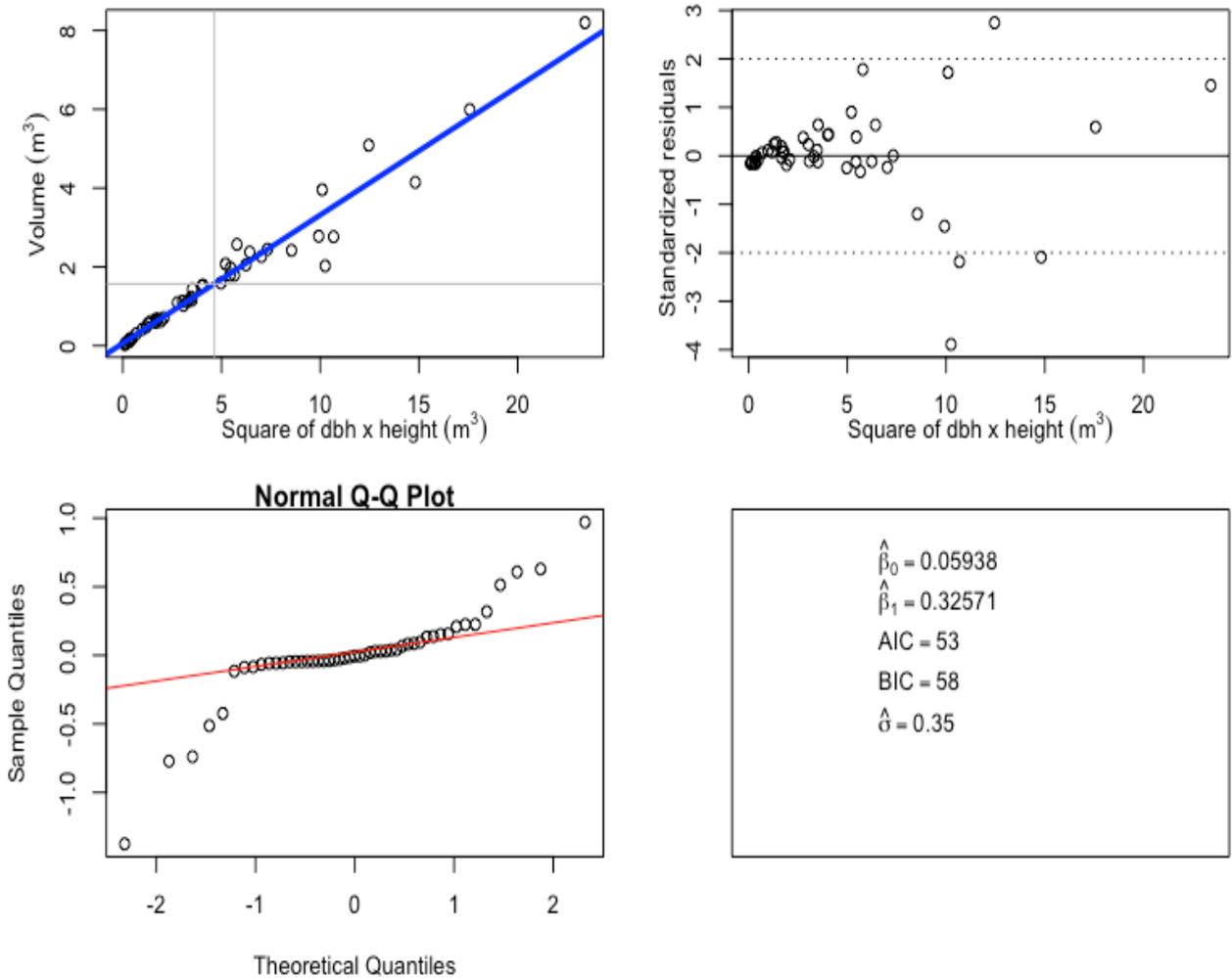
```
Model: Volume.m3 ~ DBH2H.m3
Data: NULL
      AIC      BIC    logLik
52.55197 58.10242 -23.27599
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0593814	0.06966395	0.852398	0.3983
DBH2H.m3	0.3257097	0.01035521	31.453695	0.0000

**Plot of Model 9**

J\_recurva:Model 9: (Volume ~ dbh<sup>2</sup>\*H)



7.10 Model 10 – Volume with square of diameter at breast height \* height (DBH2H) as predictor, with varFixed

```
> jr.m10 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,
               na.action=na.omit, weights = varFixed(~DBH2H.m3))
> summary(jr.m10)
```

Generalized least squares fit by REML

```
Model: Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints
Data: NULL
      AIC      BIC    logLik
2.240306 9.554872 2.879847
```

Variance function:

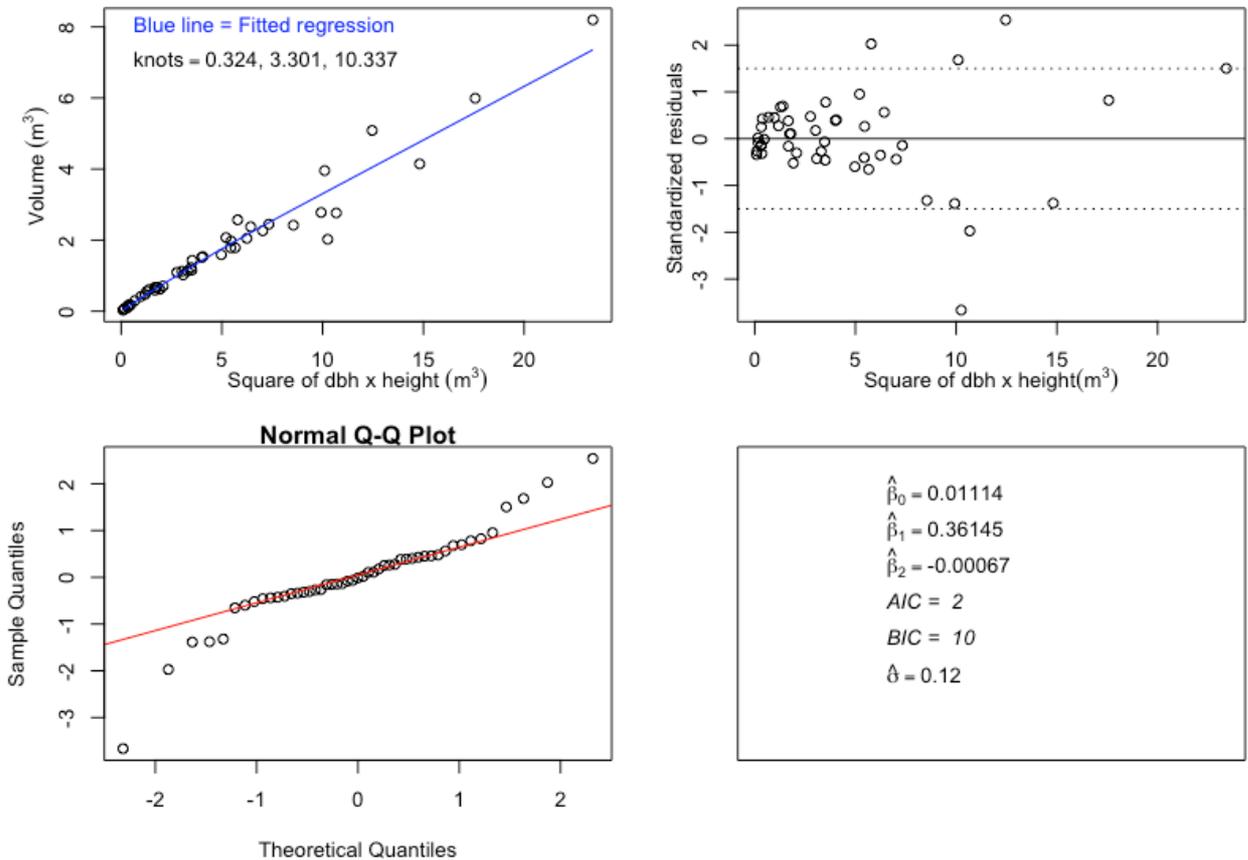
```
Structure: fixed weights
Formula: ~DBH2H.m3
```

Coefficients:

	Value	Std. Error	t-value	p-value
(Intercept)	0.0111428	0.017918495	0.621860	0.5371
DBH2H.m3	0.3614476	0.017811738	20.292663	0.0000
DBH2H.m3.splinepoints	-0.0006665	0.000370006	-1.801289	0.0782

**Plot of Model 10**

**J\_recurva:Model 10: (Volume ~ dbh<sup>2</sup>\*H), Cubic Spline with varFixed**



7.11 Model 11– Volume with square of diameter at breast height \* height (DBH2H) as predictor, with varPower

```
> jr.m11 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,
  na.action=na.omit, weights = varPower(form =
  ~DBH2H.m3))
> summary(jr.m11)
```

Generalized least squares fit by REML

```
Model: Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints
Data: NULL
      AIC      BIC    logLik
-40.76813 -31.62492 25.38407
```

Variance function:

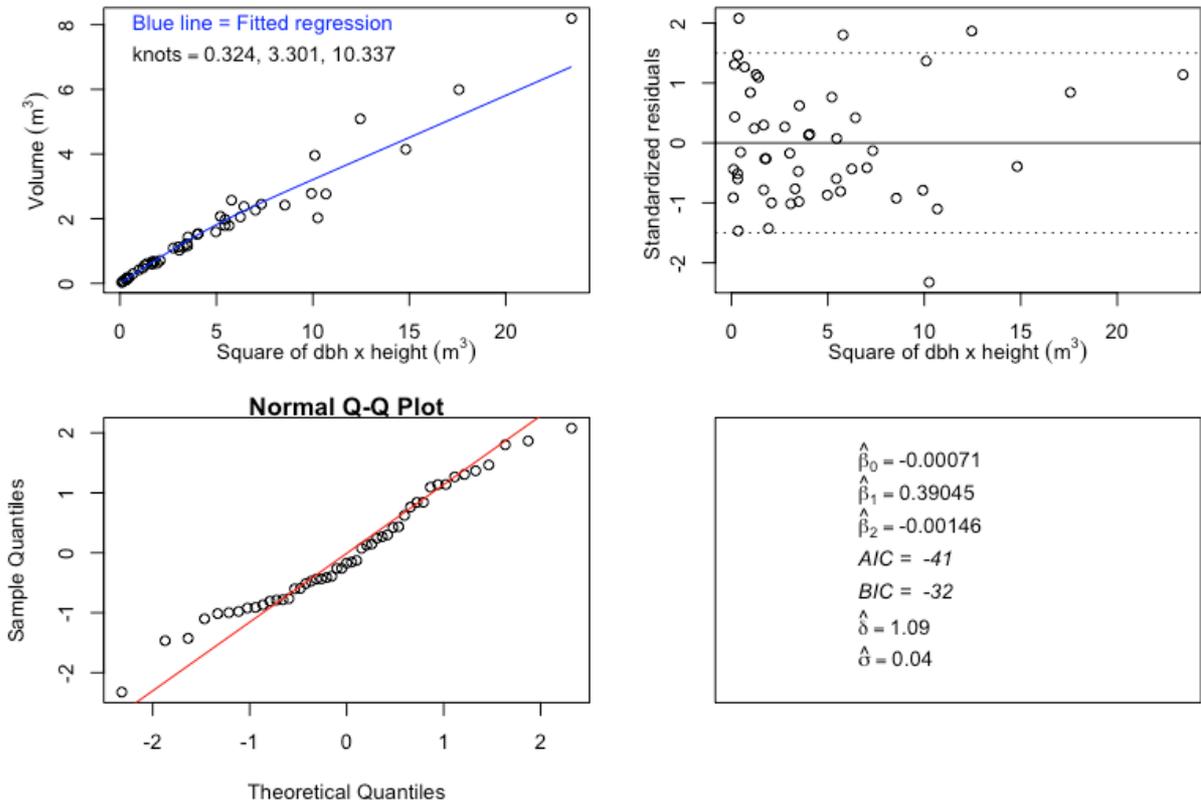
```
Structure: Power of variance covariate
Formula: ~DBH2H.m3
Parameter estimates:
  power
1.085881
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.0007136	0.002719578	-0.26240	0.7942
DBH2H.m3	0.3904517	0.010543278	37.03323	0.0000
DBH2H.m3.splinepoints	-0.0014559	0.000382813	-3.80329	0.0004

**Plot of Model 11**

**J\_recurva:Model 11: (Volume ~ dbh<sup>2</sup>\*H), Cubic Spline with varPower**



7.12 Model 12 –Volume with square of diameter at breast height \* height (DBH2H) as predictor, with varConstPower

```
> jr.m12 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,
  na.action=na.omit, weights = varConstPower(form =
  ~DBH2H.m3))
> summary(jr.m12)
```

Generalized least squares fit by REML

```
Model: Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints
Data: NULL
      AIC      BIC    logLik
-39.72691 -28.75507 25.86346
```

Variance function:

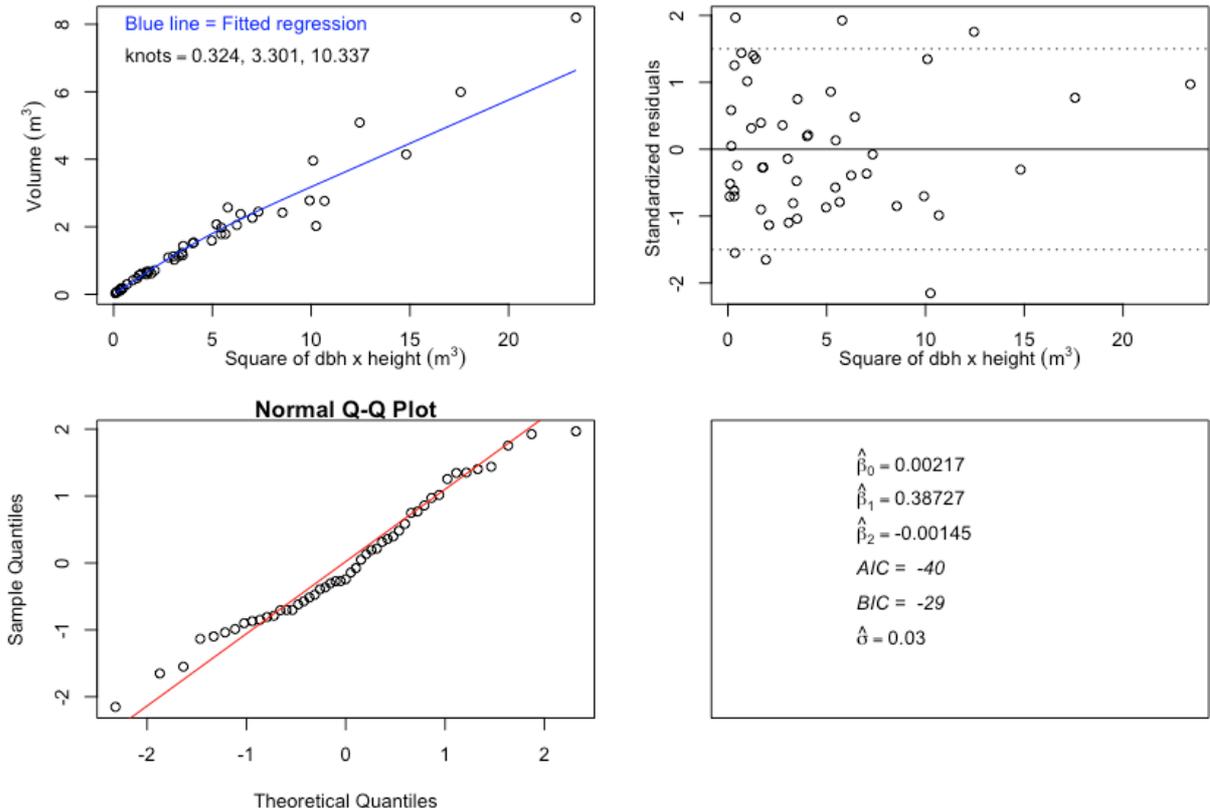
```
Structure: Constant plus power of variance covariate
Formula: ~DBH2H.m3
Parameter estimates:
      const      power
0.2218998 1.2699031
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0021707	0.004451806	0.48759	0.6282
DBH2H.m3	0.3872668	0.010263771	37.73144	0.0000
DBH2H.m3.splinepoints	-0.0014512	0.000404906	-3.58401	0.0008

**Plot of Model 12**

J\_recurva:Model 12: (Volume ~ dbh^2\*H), Cubic Spline with varConstPower



7.13 Model 13 – Volume with basal area \* height (BAH) as predictor

```
> jr.m13 <- gls(Volume.m3 ~ BAH.m3)
> summary(jr.m13)
```

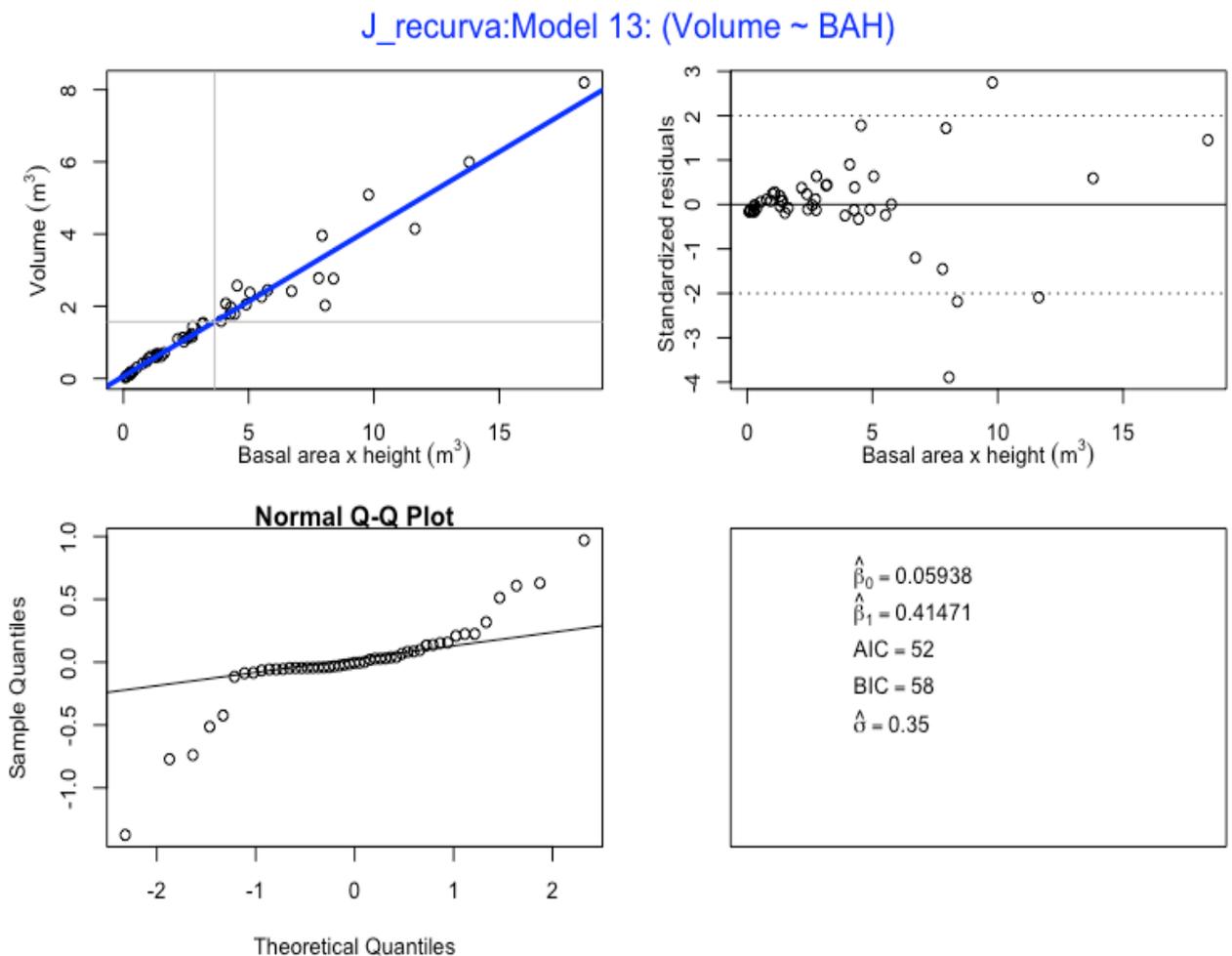
Generalized least squares fit by REML

Model: Volume.m3 ~ BAH.m3  
 Data: NULL  
           AIC          BIC      logLik  
 52.06884 57.61929 -23.03442

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0593814	0.06966395	0.852398	0.3983
BAH.m3	0.4147065	0.01318467	31.453695	0.0000

**Plot of Model 13**



7.14 Model 14 – Volume with basal area \* height (BAH) as predictor, with varFixed

```
> jr.m14 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
                na.action=na.omit, weights = varFixed(~BAH.m3))
> summary(jr.m14)
```

Generalized least squares fit by REML  
 Model: Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints  
 Data: NULL  
 AIC        BIC     logLik  
 0.3077907 7.622356 3.846105

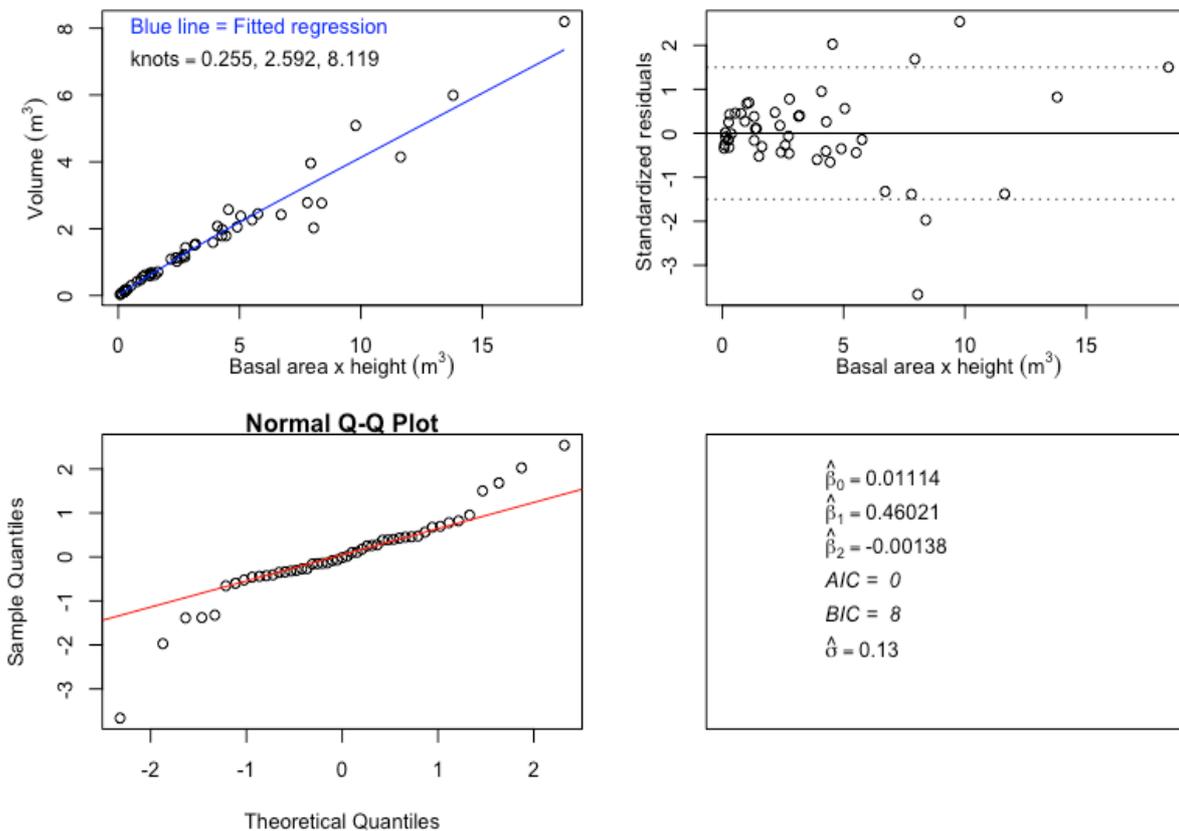
Variance function:  
 Structure: fixed weights  
 Formula: ~BAH.m3

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0111428	0.01791849	0.621860	0.5371
BAH.m3	0.4602094	0.02267861	20.292663	0.0000
BAH.m3.splinepoints	-0.0013757	0.00076373	-1.801289	0.0782

Plot of Model 14

J\_recurva:Model 14: (Volume ~ BAH), Cubic spline with varFixed



7.15 Model 15– Volume with basal area \* height (BAH) as predictor, with varPower

```
> jr.m15 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
  na.action=na.omit, weights = varPower(form =
  ~BAH.m3))
> summary(jr.m15)
```

Generalized least squares fit by REML  
 Model: Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints  
 Data: NULL  
 AIC BIC logLik  
 -42.70065 -33.55744 26.35032

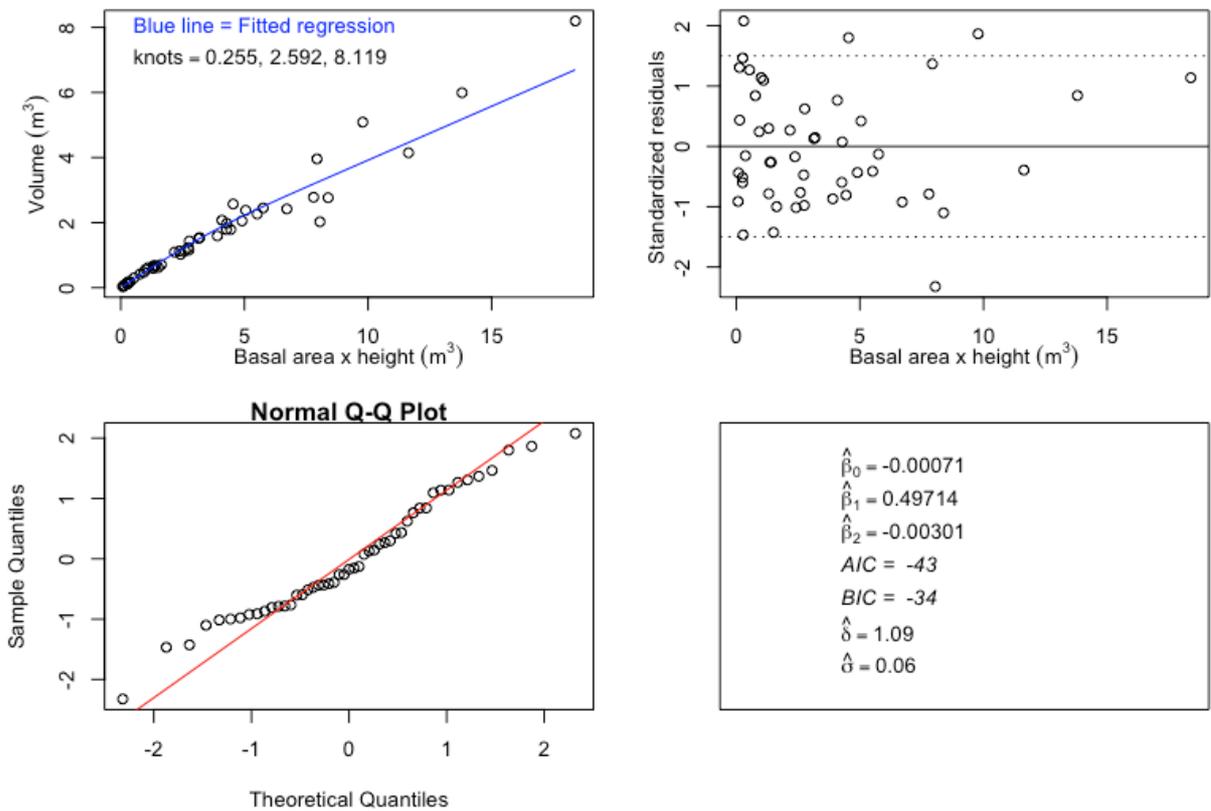
Variance function:  
 Structure: Power of variance covariate  
 Formula: ~BAH.m3  
 Parameter estimates:  
 power  
 1.085881

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.0007136	0.002719578	-0.26240	0.7942
BAH.m3	0.4971385	0.013424119	37.03323	0.0000
BAH.m3.splinepoints	-0.0030052	0.000790163	-3.80329	0.0004

**Plot of Model 15**

**J\_recurva:Model 15: (Volume ~ BAH), Cubic spline with varPower**



7.16 Model 16 – Volume with basal area \* height (BAH) as predictor, with varConstPower

```
> jr.m16 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
  na.action=na.omit, weights = varConstPower(form =
  ~BAH.m3))
> summary(jr.m16)
```

Generalized least squares fit by REML  
 Model: Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints  
 Data: NULL  
 AIC BIC logLik  
 -41.65943 -30.68758 26.82972

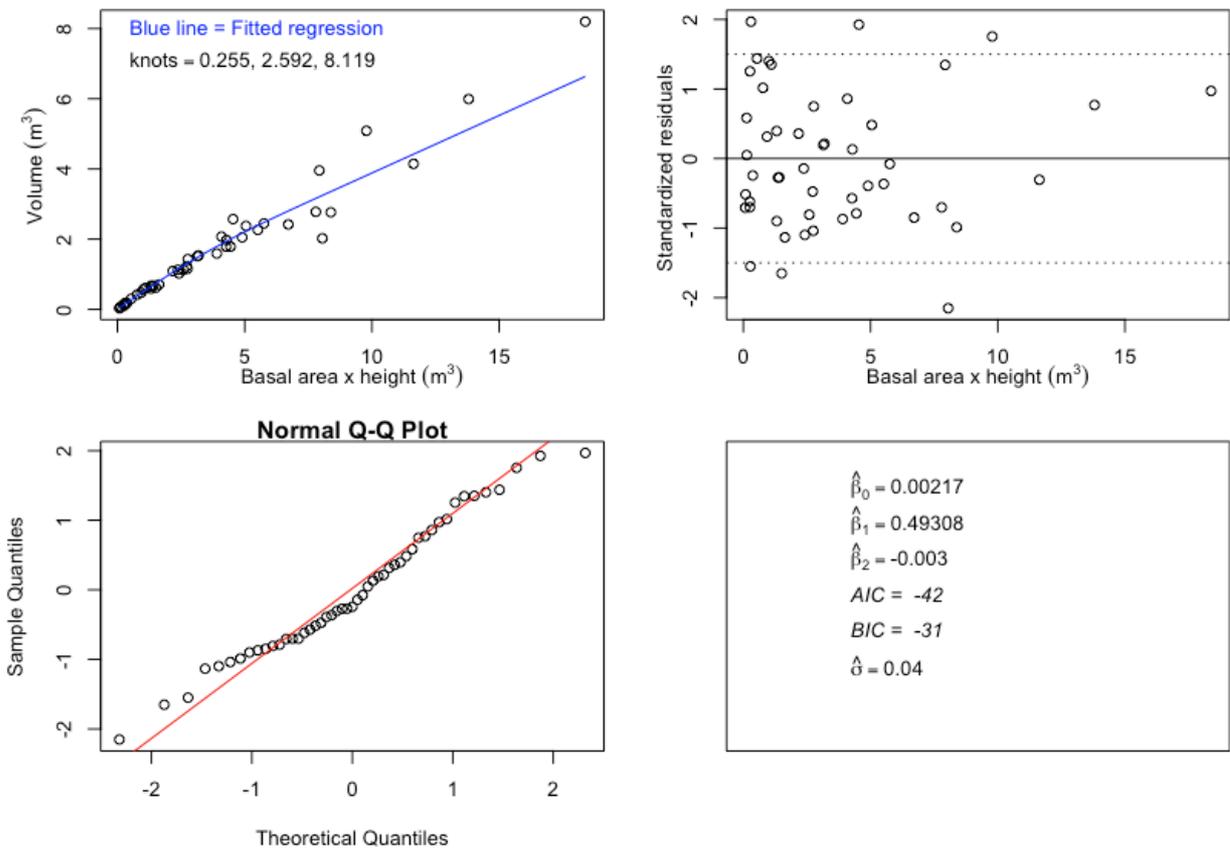
Variance function:  
 Structure: Constant plus power of variance covariate  
 Formula: ~BAH.m3  
 Parameter estimates:  
 const power  
 0.1632793 1.2699031

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0021707	0.004451806	0.48759	0.6282
BAH.m3	0.4930835	0.013068239	37.73144	0.0000
BAH.m3.splinepoints	-0.0029954	0.000835766	-3.58401	0.0008

**Plot of Model 16**

J\_recurva:Model 16: (Volume ~ BAH), Cubic spline with varConstPower



## 1. Model evaluation using AIC and BIC values

SN	Model	AIC	BIC
1	Model 1 > jr.m1 <- gls(Volume.m3 ~ DBH.cm)	126	132
2	Model 2 > jr.m2 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints, na.action=na.omit, weights = varFixed(~DBH.cm))	97	104
3	Model 3 > jr.m3 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints, na.action=na.omit, weights = varPower(form = ~DBH.cm))	42	51
4	Model 4 > jr.m4 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints, na.action=na.omit, weights = varConstPower(form = ~DBH.cm))	38	49
5	Model 5 > jr.m5 <- gls(Volume.m3 ~ BA.m2)	87	93
6	Model 6 > jr.m6<- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights = varFixed(~BA.m2))	22	29
7	Model 7 > jr.m7 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights = varPower(form = ~BA.m2))	-15	-6
8	Model 8 > jr.m8 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights = varConstPower(form = ~BA.m2))	-14	-3
9	Model 9 > jr.m9 <- gls(Volume.m3 ~ DBH2H.m3)	53	58
10	Model 10 > jr.m10 <-gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints, na.action=na.omit, weights = varFixed(~DBH2H.m3))	2	10

11	Model 11 > jr.m11 <-gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints, na.action=na.omit, weights = varPower(form = ~DBH2H.m3))	-41	-32
12	Model 12 > jr.m12 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints, na.action=na.omit, weights = varConstPower(form = ~DBH2H.m3))	-40	-29
13	Model 13 > jr.m13 <- gls(Volume.m3 ~ BAH.m3)	52	58
14	Model 14 > jr.m14 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints, na.action=na.omit, weights = varFixed(~BAH.m3))	0.3	8
15	Model 15 > jr.m15 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints, na.action=na.omit, weights = varPower(form = ~BAH.m3))	-43	-34
16	Model 16 > jr.m16 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints, na.action=na.omit, weights = varConstPower(form = ~BAH.m3))	-42	-31

## 8. Selected Models

The best fitting models have been selected based on Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) values of the fitted models. The BIC value was mainly relied upon as it imposes a stronger penalty for the number of parameters in the model that need to be estimated. Smaller the values of AIC and BIC, better the fit of the model. Therefore, for *Juniperus recurva*, the selected models are;

1. Model 7 (Model which doesn't use height)
 

```
jr.m7 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
              na.action=na.omit, weights = varPower(form = ~BA.m2))
```
2. Model 15 (Model which uses the height)
 

```
jr.m15 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
               na.action=na.omit, weights = varPower(form = ~BAH.m3))
```

Two models have been selected for *Juniperus recurva*, one without height ( $X_1 = BA$  which is model 7) and one with the height ( $X_1 = BAH$ , which is Model 15) as predictor or explanatory variable. Both the models have been fitted with natural (restricted) cubic spline function within a linear model framework. Although, nonlinear models are more flexible, they are more complicated than the linear models. The complications involved and amount of time and efforts spent on fitting nonlinear models often fail to justify by the improvements in the models. Moreover, the models fitted with natural (restricted) cubic spline functions perform well and track the curvilinearity better than nonlinear functions that were examined.

## 9. Demonstration of use of the selected best fit model

In general, the natural spline predictor with knots represented by  $t_1$ ,  $t_2$  and  $t_3$  takes the following form;

$$Y = \beta_0 + \beta_1 X + \beta_2 X_s + \varepsilon \quad (8)$$

Where  $X_s$  corresponds to value in  $X$  as follows:

$$X_s = g(X) = (X - t_1)_+^3 - (X - t_2)_+^3 \frac{(t_3 - t_1)}{(t_3 - t_2)} + (X - t_3)_+^3 \frac{(t_2 - t_1)}{(t_3 - t_2)} \quad (9)$$

and the value of the positive part functions depend on the values of the knots as follows;

$$(X - t_1)_+^3 = (X - t_1)_+^3, \text{ if } X > t_1 \text{ and } (X - t_1)_+^3 = 0, \text{ if } X < t_1 \quad (10)$$

$$(X - t_2)_+^3 = (X - t_2)_+^3, \text{ if } X > t_2, \text{ and } (X - t_2)_+^3 = 0, \text{ if } X < t_2 \quad (11)$$

$$(X - t_3)_+^3 = (X - t_3)_+^3, \text{ if } X > t_3, \text{ and } (X - t_3)_+^3 = 0, \text{ if } X < t_3 \quad (12)$$

Where  $t_1$ ,  $t_2$  and  $t_3$  for the above models are 10<sup>th</sup>, 50<sup>th</sup> and 90<sup>th</sup> percentiles and are called knots. The values of knots differ from species and models.

To demonstrate use of the selected models for *Juniperus recurva* – model 7, the knots  $t_1$ ,  $t_2$  and  $t_3$  are 0.024, 0.119 and 0.343 as generated by the model. The model 7 has been fitted with volume as function of basal area in meter square (BA) i.e

$$BA = \pi r^2 \quad (13)$$

where in

$$r^2 = \left[ \frac{dbh}{2 \times 100} \right]^2 \quad (14)$$

Where  $r$  is radius in meters and  $dbh$  is diameter at breast height in centimeters.

Therefore, *Juniperus recurva* with diameter of 56.6 cm resulting in basal area of 0.251607014 m<sup>2</sup>, the volume can be estimated using the above equation (model 7) as below. But first the value of BA.m<sup>2</sup> has to be calculated, which is;

$$\begin{aligned} BA &= \pi r^2 = \frac{\pi * 56.6^2}{200^2} = 0.251607014 \text{ m}^2 \\ g(X) &= (X - t_1)_+^3 - (X - t_2)_+^3 \frac{(t_3 - t_1)}{(t_3 - t_2)} + (X - t_3)_+^3 \frac{(t_2 - t_1)}{(t_3 - t_2)} \\ g(BA) &= (BA - t_1)_+^3 - (BA - t_2)_+^3 \frac{(t_3 - t_1)}{(t_3 - t_2)} + (BA - t_3)_+^3 \frac{(t_2 - t_1)}{(t_3 - t_2)} \\ g(BA) &= (0.251607014 - 0.024)_+^3 - (0.251607014 - 0.119)_+^3 \frac{(0.343 - 0.024)}{(0.343 - 0.119)} + 0 \\ &= (0.227607014)_+^3 - (0.251607014 - 0.119)_+^3 \frac{(0.319)}{(0.224)} + 0 \\ &= (0.227607014)_+^3 - (0.132607014)_+^3 * 1.42410714 + 0 \\ &= 0.01179117 - 0.00233184 * 1.42410714 \\ &= 0.01179117 - 0.00332079 \\ &= 0.00847037 \end{aligned}$$

Hence, the volume predicted for this tree by the selected model (model 7) is

$$\begin{aligned} V &= \beta_0 + \beta_1 \cdot BA + \beta_2 BA \cdot m_2 + \varepsilon \\ &= -0.057974 + 9.240696 * 0.251607014 + 26.700167 * 0.00847037 \\ &= -0.057974 + 2.325024 + 0.226160 \\ &= \mathbf{2.49321 \text{ m}^3} \end{aligned}$$

Similarly, to demonstrate model 15 with  $t_1$ ,  $t_2$  and  $t_3$  of 0.255, 2.592 and 8.119 respectively, we considered this same tree but with height, i.e  $dbh = 56.6$  cm resulting in  $BA = 0.251607014$  m<sup>2</sup> and height (H) = 18.05 m.

$$\begin{aligned} BAH &= 0.251607014 \times 18.05 \\ &= 4.5415066027 \end{aligned}$$

$$\begin{aligned} g(X) &= (X - t_1)_+^3 - (X - t_2)_+^3 \frac{(t_3 - t_1)}{(t_3 - t_2)} + (X - t_3)_+^3 \frac{(t_2 - t_1)}{(t_3 - t_2)} \\ g(BAH) &= (BAH - t_1)_+^3 - (BAH - t_2)_+^3 \frac{(t_3 - t_1)}{(t_3 - t_2)} + (BAH - t_3)_+^3 \frac{(t_2 - t_1)}{(t_3 - t_2)} \end{aligned}$$

$$\begin{aligned}
&= (4.5415066027 - 0.255)_+^3 - (4.5415066027 - 2.592)_+^3 \frac{(8.119 - 0.255)}{(8.119 - 2.592)} + 0 \\
&= (4.2865066027)_+^3 - (1.9495066027)_+^3 \frac{(7.864)}{(5.527)} + 0 \\
&= (4.2865066027)_+^3 - (1.9495066027)_+^3 * 1.4228334 + 0 \\
&= 78.760868 - 7.409248 * 1.4228334 + 0 \\
&= 78.760868 - 10.542126 \\
&= 68.218742
\end{aligned}$$

Hence, the volume predicted by model 15 for this tree is;

$$\begin{aligned}
V &= \beta_0 + \beta_1 \cdot BAH.m^3 + \beta_2 BAH.m^3_2 + \varepsilon \\
&= -0.0007136 + 0.4971385 * 4.5415066027 + (-0.0030052 * 68.218742) \\
&= -0.0007136 + 2.2577577 + (-0.20501096) \\
&= 2.052033 \text{ m}^3
\end{aligned}$$

The field measured volume for this particular tree with DBH of 56.6 cm and height of 18.05 m is 2.572026 m<sup>3</sup>.

## 10. Model Performance

To assess the performance of selected models, we compared the volume predicted by selected models (7 and 15) with the volume of the tree as measured in the field. Using the equations of the selected models, volume prediction or estimation was done in R.

SN	Tree_ID	Height (in m)	DBH (in cm)	Volume in m <sup>3</sup> (Field measured) [A]	Predicted Volume Model_7 [B]	Predicted Volume Model_15 [C]	Difference (Field - Model_7) [A - B]	Difference (Field - Model_15) [A - C]
1	jre01	17.05	24	0.417693623	0.360321045	0.382327514	0.057372578	0.035366109
2	jre02	14.95	37.3	0.708499778	0.968327162	0.803543424	-0.259827384	-0.095043647
3	jre03	21.4	54	2.050983698	2.237809012	2.187049641	-0.186825315	-0.136065944
4	jre04	24.1	47.5	1.786968863	1.668044225	1.948018307	0.118924638	-0.161049444
5	jre05	26.9	63	2.765152766	3.186372212	3.384262479	-0.421219446	-0.619109713
6	jre06	28.9	71.6	4.146103863	4.246829642	4.461760943	-0.100725779	-0.31565708
7	jre08	10.3	13	0.070050339	0.064679874	0.067252321	0.005370465	0.002798017
8	jrec01	16.5	26.8	0.473646096	0.464206716	0.461081691	0.009439381	0.012564406
9	jrec02	20.3	44.4	1.51479722	1.430807599	1.490143315	0.083989621	0.024653905
10	jrec03	21.7	39	1.129432213	1.06913638	1.249629137	0.060295832	-0.120196924
11	jrec04	18.75	32	0.618633744	0.690003357	0.743042206	-0.071369613	-0.124408462
12	jrec06	23	42	1.537156974	1.262114025	1.508615824	0.275042948	0.02854115
13	jrec07	21.35	36	1.093081022	0.895181677	1.058440988	0.197899344	0.034640034
14	jrec08	13.95	15.2	0.118677239	0.109706183	0.125129284	0.008971055	-0.006452045
15	jrec09	22.5	47	1.590720282	1.628287138	1.803608045	-0.037566856	-0.212887763
16	jrec10	22.6	61.5	2.419587798	3.015983633	2.826587073	-0.596395835	-0.406999274
17	jrec11	10.95	18	0.117363299	0.177173152	0.137810802	-0.059809853	-0.020447503
18	jrec12	21.1	57.7	2.263579361	2.605920912	2.411249391	-0.34234155	-0.14767003
19	jrec13	21.4	50.5	1.974207923	1.918771471	1.954099516	0.055436452	0.020108407
20	jrec14	22.42	27.3	0.670486573	0.484029323	0.648155277	0.18645725	0.022331296
21	jrec17	7.38	12.4	0.041818245	0.053619252	0.043592856	-0.011801007	-0.001774611
22	jrec19	19.4	54	1.786607484	2.237809012	2.014454826	-0.451201529	-0.227847343
23	jrec20	8	14.3	0.071023339	0.090437187	0.063161169	-0.019413848	0.007862171
24	jrec21	9.6	18.4	0.118562161	0.187740639	0.126190063	-0.069178478	-0.007627902
25	jrec22	8.95	10.3	0.033320581	0.019022151	0.03636	0.01429843	-0.003039419
26	jrec23	11.15	20.6	0.181071786	0.250032282	0.18402829	-0.068960496	-0.002956504
27	jrec24	20.45	41.2	1.230721499	1.208732524	1.309298936	0.021988974	-0.078577438
28	jrec25	20.94	38	1.126660221	1.009112229	1.151281841	0.117547992	-0.024621621
29	jrec26	19.31	30	0.653205241	0.597929183	0.673742972	0.055276058	-0.02053773
30	jrec27	15	45.3	1.023007594	1.497417102	1.170755968	-0.474409508	-0.147748373
31	jrec28	21.66	68.8	2.024214973	3.886569086	3.273892377	-1.862354113	-1.249677404
32	jrec29	15.78	20.8	0.301696092	0.256046453	0.265783244	0.045649639	0.035912848
33	jrec30	16.2	29.4	0.61192477	0.571603031	0.544209825	0.040321739	0.067714945
34	jrec31	19.11	58	2.377623807	2.637134573	2.24165986	-0.259510766	0.135963947

35	jrec32	22.58	66.3	2.778846934	3.577050357	3.188683001	-0.798203423	-0.409836067
36	jrwc01	32.4	85	8.197200354	6.169868817	6.698743947	2.027331537	1.498456406
37	jrwc02	25.6	53.5	2.447379968	2.190484663	2.495599783	0.256895305	-0.048219815
38	jrwc03	24.4	38	1.432295093	1.009112229	1.327361836	0.423182864	0.104933257
39	jrwc04	28.6	66	5.087699119	3.540677905	3.848033575	1.547021213	1.239665544
40	jrwc05	24.6	46	2.073262676	1.550501665	1.876779961	0.522761011	0.196482715
41	jrwc06	19.9	29	0.590300189	0.554377835	0.649168886	0.035922354	-0.058868696
42	jrwc08	9.6	18.6	0.147960361	0.193111669	0.128963831	-0.045151309	0.01899653
43	jrwo1	28.3	78.8	5.99243998	5.239195714	5.179448841	0.753244266	0.812991139
44	jrwo2	16.83	27.6	0.562648318	0.496110896	0.498584317	0.066537422	0.064064001
45	jrwo3	11.15	18.5	0.17963732	0.190418876	0.148285903	-0.010781557	0.031351417
46	jrwo4	16	33.5	0.674547272	0.763558794	0.695747862	-0.089011522	-0.02120059
47	jrwo5	17.6	44.6	1.155740271	1.445450634	1.319590204	-0.289710363	-0.163849933
48	jrwo6	24.28	64.5	3.957572555	3.361338547	3.234434851	0.596234007	0.723137704
49	jrwo8	18.05	56.6	2.572025692	2.493210354	2.052033216	0.078815338	0.519992476
				<b>76.89783657</b>	<b>75.7613784</b>	<b>76.09167942</b>	<b>1.136458166</b>	<b>0.806157147</b>

From the above table, the difference [A-B] provides difference between the volume measured in the field (actual volume) and the volume predicted by model 7. The figures with negative (-) indicates that the volume has been over-predicted by the model 7 vis-à-vis actual volume of the particular tree. And the figures without negative (-) sign indicates the under prediction of volume by the model 7.

Similarly, the difference [A-C] is the difference between the actual volume and the volume predicted by the model 15. Same explanation is applicable here – the figures with negative sign indicates overprediction of volume by the model and vice-versa, while those figures without (-) are under prediction of volume by the model 15.

Summation of the figures in the difference column results in 1.136458166 and 0.806157147 for model 7 and model 15 respectively. These indicate that the model 7 under predicts total volume for 49 trees by 1.136458166 m<sup>3</sup>, while the model 15 under predicts the total volume of 49 trees by 0.806157147 m<sup>3</sup>. Therefore, looking this, one may be inclined to conclude that overall, model 15 predicts better than model 7.

## 11. Limitations of the model

The model has the following limitations;

1. The modeling has been done based on only 49 sample trees. The model can be further improved by increasing the number of samples.
2. The diameter for the sample trees ranges between minimum of 10.30 cm to 85 cm (over bark). Thus, the model prediction for trees above 85 cm should be done with caution.

## 12. Conclusion

The model 15 that uses the height performs slightly better than the model 7 that doesn't use the height, as empirically shown above. This further reinforces and confirms the observations made by Professor Timothy Gordon Gregoire and Mr. Yograj Chettri while modeling conifer species for biomass estimation. They too observed that in conifers, the models fitted with height as predictors predicted the biomass better than those models that didn't use height as predictor variable.

This therefore, leads us to confidently conclude that the best model for *Juniperus recurva*, out of 16 models fitted above, is model 15. However, since the models have been developed using different predictor variables – model 7 (fitted without height as predictor), while model 15 (fitted with height as predictor) variables, we considered two best fit models for *Juniperus recurva*,

1. Model 7: the best fit model which doesn't use height
2. Model 15: the best fit model which uses height

### 13. Acknowledgement

We would like to express our appreciation to the biomass equation development team led by Mr. Yograj Chettri, Research Officer at UWICER, formerly RDC who collected data (diameter and height) for developing volume equations as part of field work for biomass equation development exercise.

Immense gratitude is also due to Professor Timothy Gordon Gregoire, School of Forestry and Environmental Studies (FES), Yale University who has been a guide and mentor as we worked on this assignment.

Thanks are also due to our Director, Mr. Lobzang Dorji and other colleagues working in FRMD for their support and advice.

Had it not been for the immense support, guidance and encouragement that the abovementioned people have generously provided to us, this task would have remained incomplete. Thanks a lot!

## 14. References

1. Lee, D., Seo, Y., & Choi, J. (2017). Estimation and validation of stem volume equations for *Pinus densiflora*, *Pinus koraiensis*, and *Larix kaempferi* in South Korea. *Forest Science and Technology*, 13(2), 77-82.
2. Umunay, P., Gregoire, T., & Ashton, M. (2017). Estimating biomass and carbon for *Gilbertiodendron dewevrei* (De Wild) Leonard, a dominant canopy tree of African tropical Rainforest: Implications for policies on carbon sequestration. *Forest Ecology and Management*, 404, 31-44.
3. White, J. C., Coops, N. C., Wulder, M. A., Vastaranta, M., Hilker, T., & Tompalski, P. (2016). Remote sensing technologies for enhancing forest inventories: A review. *Canadian Journal of Remote Sensing*, 42(5), 619-641.
4. Mohammadi, J., Shataee, S., & Babanezhad, M. (2011). Estimation of forest stand volume, tree density and biodiversity using Landsat ETM+ Data, comparison of linear and regression tree analyses. *Procedia Environmental Sciences*, 7, 299-304.
5. Fagan, M., & DeFries, R. (2009). Measurement and Monitoring of the World's Forests. *Resources for the Future*, 129.
6. Feng, Z. K., Yang, B.G., Luo, X., Han, G.S., Guo, X.X., (2008). Experiment of estimating forest stand volume with LiDAR technology. *The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences.*, XXXVII.
7. McRoberts, R. E., & Tomppo, E. O. (2007). Remote sensing support for national forest inventories. *Remote Sensing of Environment*, 110(4), 412-419.
8. Westfall, J. A., & Patterson, P. L. (2007). Measurement variability error for estimates of volume change. *Canadian Journal of Forest Research*, 37(11), 2201-2210.
9. Lu, D. (2006). The potential and challenge of remote sensing-based biomass estimation. *International Journal of Remote Sensing*, 27(7), 1297-1328.
10. Sadiq, R. A. (2006). A new approach to log volume estimation. *Southern Journal of Applied Forestry*, 30(1), 30-39.
11. Hyypä, J., Mielonen, T., Hyypä, H., Maltamo, M., Yu, X., Honkavaara, E., & Kaartinen, H. (2005). Using individual tree crown approach for forest volume extraction with aerial images and laser point clouds.
12. Patterson, D. W., & Doruska, P. F. (2004). A new and improved modification to Smalian's equation for butt logs. *Forest Products Journal*, 54(4), 69.
13. Eerikäinen, K. (2001). Stem volume models with random coefficients for *Pinus kesiya* in Tanzania, Zambia, and Zimbabwe. *Canadian Journal of Forest Research*, 31(5), 879-888.
14. Gregoire, T. G., & Schabenberger, O. (1996). Nonlinear mixed-effects modeling of cumulative bole volume with spatially correlated within-tree data. *Journal of Agricultural, Biological, and Environmental Statistics*, 107-119.
15. Bi, H. (1994). Volume equations for six Eucalyptus species on the south-east tablelands of New South Wales: Research Division State Forests of New South Wales.
16. Laumans, P. (1994). Height-diameter functions from PIS for country-level site classification and local volume table selection. Thimphu.
17. Biging, G. S. (1988). Estimating the accuracy of volume equations using taper equations of stem profile. *Canadian Journal of Forest Research*, 18(8), 1002-1007.
18. Reed, D. D., & Byrne, J. C. (1985). A simple, variable form volume estimation system. *The Forestry Chronicle*, 61(2), 87-90.
19. Avery, T.E., and Burkhart, H.E. (1983). *Forest Measurements*, Third Edition. McGraw-Hill, Inc.
20. Sadiq, R. A., & Smith, V. G. (1983). Estimation of individual tree volumes with age and diameter. *Canadian Journal of Forest Research*, 13(1), 32-39.
21. Cochran, P. (1982). Estimating wood volumes for Douglas-fir and white fir from outside bark measurements. *Forest Science*, 28(1), 172-174.

22. Bredenkamp, B. (1982). Volume regression equations for *Eucalyptus grandis* on the coastal plain of Zululand. *South African Forestry Journal*, 122(1), 66-69.
23. Cao, Q. v., & Burkhart, H. E. (1980). Cubic-foot volume of loblolly pine to any height limit. *Southern Journal of Applied Forestry*, 4(4), 166-168.
24. Cao, Q. V., Burkhart, H. E., & Max, T. A. (1980). Evaluation of two methods for cubic-volume prediction of loblolly pine to any merchantable limit. *Forest Science*, 26(1), 71-80.
25. Goulding, C. (1979). Cubic spline curves and calculation of volume of sectionally measured trees. *NZJ For. Sci*, 9(1), 89-99.
26. Burkhart, H. E. (1977). Cubic-foot volume of loblolly pine to any merchantable top limit. *Southern Journal of Applied Forestry*, 1(2), 7-9.
27. Heger, L. (1965). A trial of Hohenadl's method of stem form and stem volume estimation. *The Forestry Chronicle*, 41(4), 466-475.

15. Annexure – Dataset for *Juniperus recurva*

SN	Tree_ID	Height.m	DBH.cm	Volume.m3	BA.m2	BAH.m3	DBH2H.m3
1	jre01	17.05	24	0.41769362	0.04523893	0.77132383	0.98208
2	jre02	14.95	37.3	0.70849978	0.10927166	1.63361133	2.07997855
3	jre03	21.4	54	2.0509837	0.2290221	4.90107304	6.24024
4	jre04	24.1	47.5	1.78696886	0.17720546	4.2706516	5.4375625
5	jre05	26.9	63	2.76515277	0.31172453	8.38538989	10.67661
6	jre06	28.9	71.6	4.14610386	0.40263908	11.6362694	14.8157584
7	jre08	10.3	13	0.07005034	0.01327323	0.13671426	0.17407
8	jrec01	16.5	26.8	0.4736461	0.05641044	0.93077222	1.185096
9	jrec02	20.3	44.4	1.51479722	0.15483025	3.14305412	4.0018608
10	jrec03	21.7	39	1.12943221	0.11945906	2.59226162	3.30057
11	jrec04	18.75	32	0.61863374	0.08042477	1.50796447	1.92
12	jrec06	23	42	1.53715697	0.13854424	3.18651743	4.0572
13	jrec07	21.35	36	1.09308102	0.1017876	2.1731653	2.76696
14	jrec08	13.95	15.2	0.11867724	0.01814584	0.25313446	0.3223008
15	jrec09	22.5	47	1.59072028	0.17349445	3.90362522	4.97025
16	jrec10	22.6	61.5	2.4195878	0.29705722	6.71349318	8.547885
17	jrec11	10.95	18	0.1173633	0.0254469	0.27864356	0.35478
18	jrec12	21.1	57.7	2.26357936	0.26148183	5.51726651	7.0248019
19	jrec13	21.4	50.5	1.97420792	0.20029617	4.28633797	5.457535
20	jrec14	22.42	27.3	0.67048657	0.05853494	1.31235335	1.67094018
21	jrec17	7.38	12.4	0.04181825	0.01207628	0.08912296	0.11347488
22	jrec19	19.4	54	1.78660748	0.2290221	4.44302883	5.65704
23	jrec20	8	14.3	0.07102334	0.01606061	0.12848486	0.163592
24	jrec21	9.6	18.4	0.11856216	0.02659044	0.25526823	0.3250176
25	jrec22	8.95	10.3	0.03332058	0.00833229	0.07457399	0.09495055
26	jrec23	11.15	20.6	0.18107179	0.03332916	0.37162009	0.4731614
27	jrec24	20.45	41.2	1.2307215	0.13331663	2.726325	3.4712648
28	jrec25	20.94	38	1.12666022	0.11341149	2.3748367	3.023736
29	jrec26	19.31	30	0.65320524	0.07068583	1.36494347	1.7379
30	jrec27	15	45.3	1.02300759	0.16117077	2.41756158	3.078135
31	jrec28	21.66	68.8	2.02421497	0.37176351	8.05239759	10.252631
32	jrec29	15.78	20.8	0.30169609	0.03397947	0.53619598	0.68270592
33	jrec30	16.2	29.4	0.61192477	0.06788668	1.09976415	1.4002632
34	jrec31	19.11	58	2.37762381	0.26420794	5.04901377	6.428604
35	jrec32	22.58	66.3	2.77884693	0.34523669	7.79544435	9.92546802
36	jrwc01	32.4	85	8.19720035	0.56745017	18.3853856	23.409
37	jrwc02	25.6	53.5	2.44737997	0.22480059	5.75489509	7.32736
38	jrwc03	24.4	38	1.43229509	0.11341149	2.76724047	3.52336
39	jrwc04	28.6	66	5.08769912	0.34211944	9.78461598	12.45816

Merchantable\_volume\_equation\_ *Juniperus recurva* : 35

40	jrwc05	24.6	46	2.07326268	0.16619025	4.08828018	5.20536
41	jrwc06	19.9	29	0.59030019	0.06605199	1.31443451	1.67359
42	jrwc08	9.6	18.6	0.14796036	0.02717163	0.26084769	0.3321216
43	jrwc01	28.3	78.8	5.99243998	0.48768828	13.8015782	17.5727152
44	jrwc02	16.83	27.6	0.56264832	0.05982849	1.0069135	1.28204208
45	jrwc03	11.15	18.5	0.17963732	0.02688025	0.29971481	0.38160875
46	jrwc04	16	33.5	0.67454727	0.08814131	1.41026094	1.7956
47	jrwc05	17.6	44.6	1.15574027	0.15622826	2.74961739	3.5009216
48	jrwc06	24.28	64.5	3.95757255	0.32674527	7.93337518	10.101087
49	jrwc08	18.05	56.6	2.57202569	0.25160701	4.5415066	5.7824258